Secrets of the Implied Volatility Surface

Inferring Rates, Dividends, Events, and Risk-Neutral Densities from Options Prices

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Implied Vols and Surfaces

- **Implied vol = "the wrong number to plug into the wrong formula to get the right price" !!?** YES and NO...
- **Implied volatility surfaces** (& implied borrow/forward curves) are the standard approach to summarizing the vanilla options market in an intuitive and compact manner.
- They provide the fundamental building block for the trading of vanillas (listed and OTC), as well as flow derivatives and exotics.
- There are many quant problems facing options and derivatives trading desks, and the problem of **constructing sensible, arbitrage-free volatility surfaces from options market prices** (bids/asks) is one of the hardest, esp. in real time.
- This issue already exist for European-style options (SPX, SX5E, DAX, etc).
- But the topic gets a lot more interesting & complicated for American options.

Implied Vols and Surfaces

- For European options (without divs) only integrated rates and variances matter.
  - Cash dividend modeling is relatively minor issue for Euro options (unless stochastic divs…).
- But **American** options are really path-dependent exotics and a lot of extra complications arise (esp. for ETFs, stocks, esp. with dividends):
  - Need to choose proper cash dividend and borrow cost modeling. Then:
  - Even in BS: Besides rate term-structure, proper choice of “vol time” (aka “trading time”), including “events” affects early exercise premia (EEP), and all details matter, incl. “settlement”.
  - Beyond BS: Local vol? Stochastic LV? Hacks?
- There are subtleties in “de-Americanization”, but if in doubt think of “implied vol surfaces” as summarizing European options prices in a convenient and intuitive manner (whether they are listed/traded or not).

Other Inputs for Pricing: Forwards, Rates, Divs

- To price European options of any expiry and payoff we need:
  - A discount rate curve: Term rates r(T)
  - A forward curve: F(T)   (also divs in some models of pricing Euros…)
  - An implied vol surface (IVS aka VS).
- Convenient to think of the **forward curve** in terms of **rates** and **divs** (0 if Fut, FX)
  - In the **American** case, the **rates are primary**, i.e. needed for proper EEP calculation.
  - The forward grows with the “drift” between divs, and jumps down by the div amount at ex-div dates.   “drift” = fundingRate – borrowRate = b – q:

$$F_T = f_p(T)\left(S_0 - \sum_{i:t_i \le T} \frac{d_i}{f_p(t_i)}\right) \qquad f_p(t) := \exp\left(\int_0^t \left(b(t') - q(t')\right)\, dt'\right)$$

NB: Sometimes b=r is used, since the difference can be absorbed into the (implied) borrow rate q.

A bit more about the (Implied) Borrow Cost

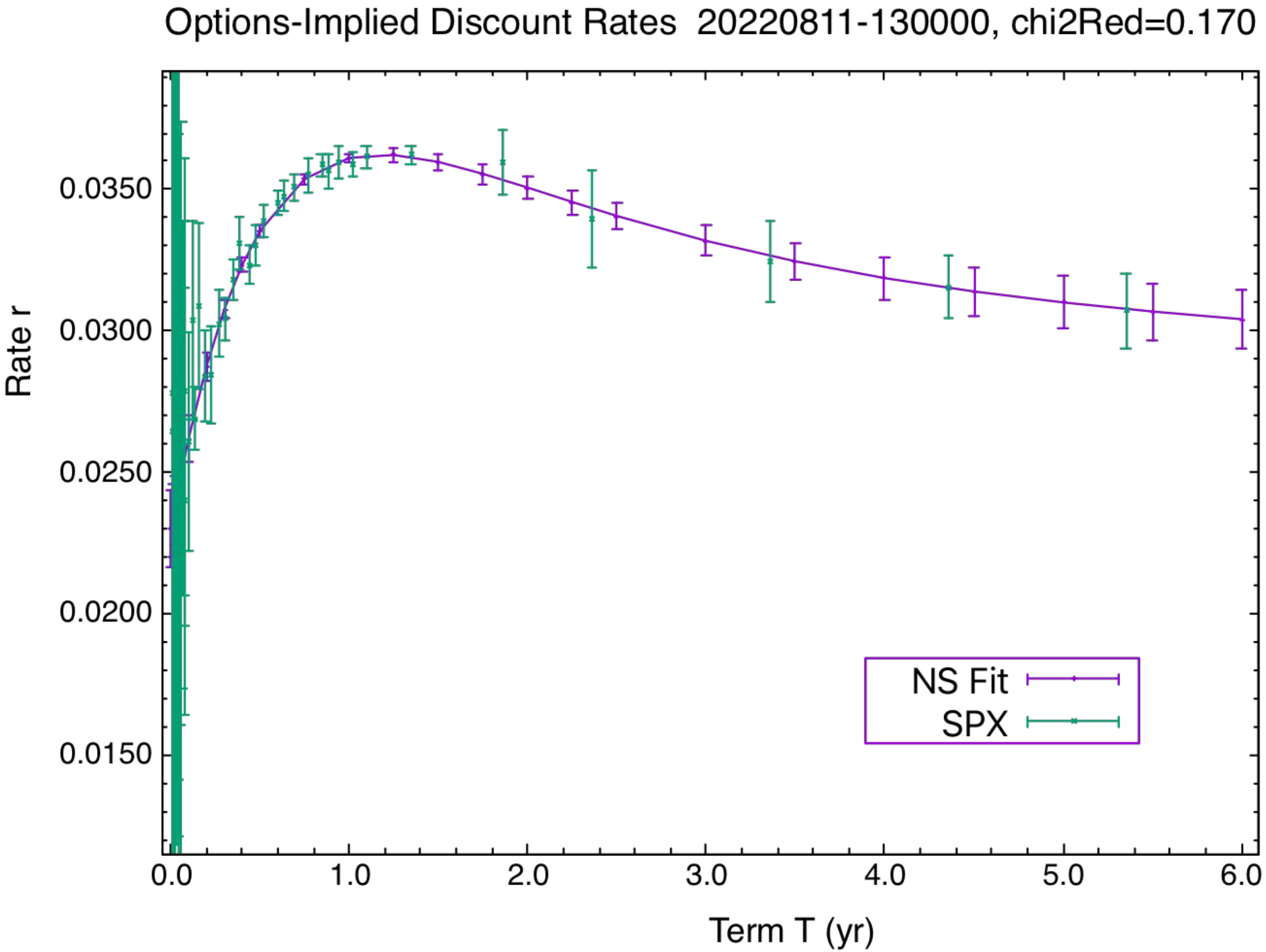
- What is the “borrow fee” that an agent bank/broker might charge a prime broker/hedge fund if the latter wants to borrow a stock or ETF (e.g. for shorting)?
- “Stock loan” aka “securities lending” mechanics is complicated...
- Mostly have to know that it acts like a div yield: If you own the stock you (can) get paid.
- Naively... the real world is much more messy: The “implied borrow cost”, which is a function of expiry, q(T), can be very different from the (overnight or term) borrow fee.
- The (implied) borrow cost is something of a “fudge factor” used by market participants to balance supply & demand, to absorb modeling errors, etc.
- It has a bid/ask. You might want to use your own borrow in pricing (depending on your position, PB rates, etc).
- But, to understand market consensus, q(T) has to be **implied** (in quasi real time).

“SPIBOR” — SPX Implied Discount Rates

- What discount curve should one use for options pricing and trading?
  - Depends… but for implying borrows, vols, etc, use market consensus.
- European put-call-parity (PCP) for a given term:
  - $$C - P = DF \cdot F - DF \cdot K$$
- To imply the discount factor for a given term T, DF = DF(T), we need a robust linear regression across many strikes K.
- For further robustness, can smooth rates across T via a term-structure fit.

SPIBOR — Even the Fed cares now!

- Fed (-associated) economists have written a number of papers about SPIBOR in the last few years.
- Why does the Fed care?
  - The Fed needs to know what's going on...
  - **Treasury, SOFR, etc rates are NOT risk-free rates!**
  - They can be lower than risk-free ("convenience yield"), or higher ("default risk").
  - Usually they are a bit lower, by 20 – 40 bps (almost flat).
  - SPX options market makers should be using close to risk-free rates ("box rates") due to margin requirements at exchange and OCC level.

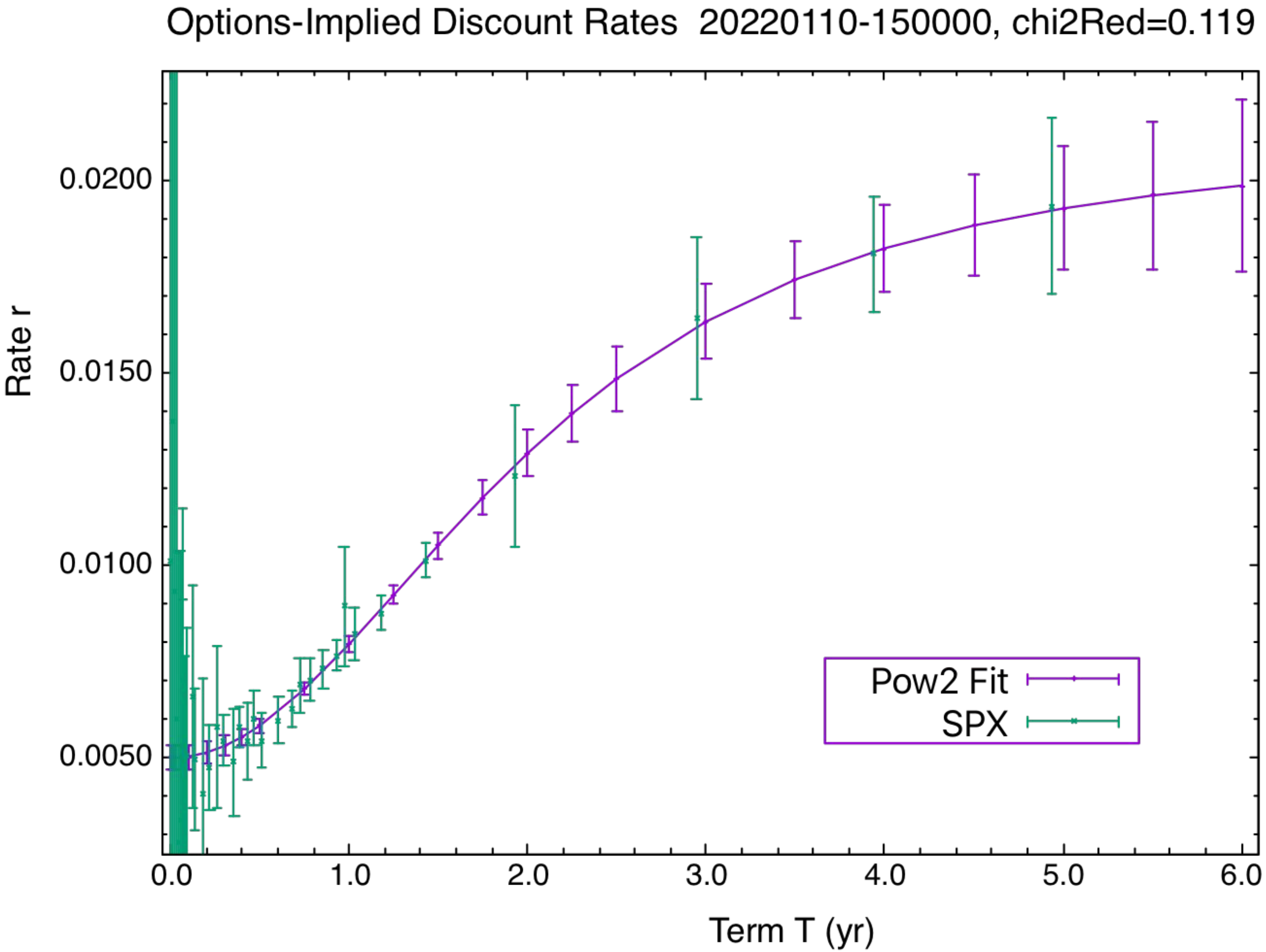


What discount rates should I use?

SPIBOR

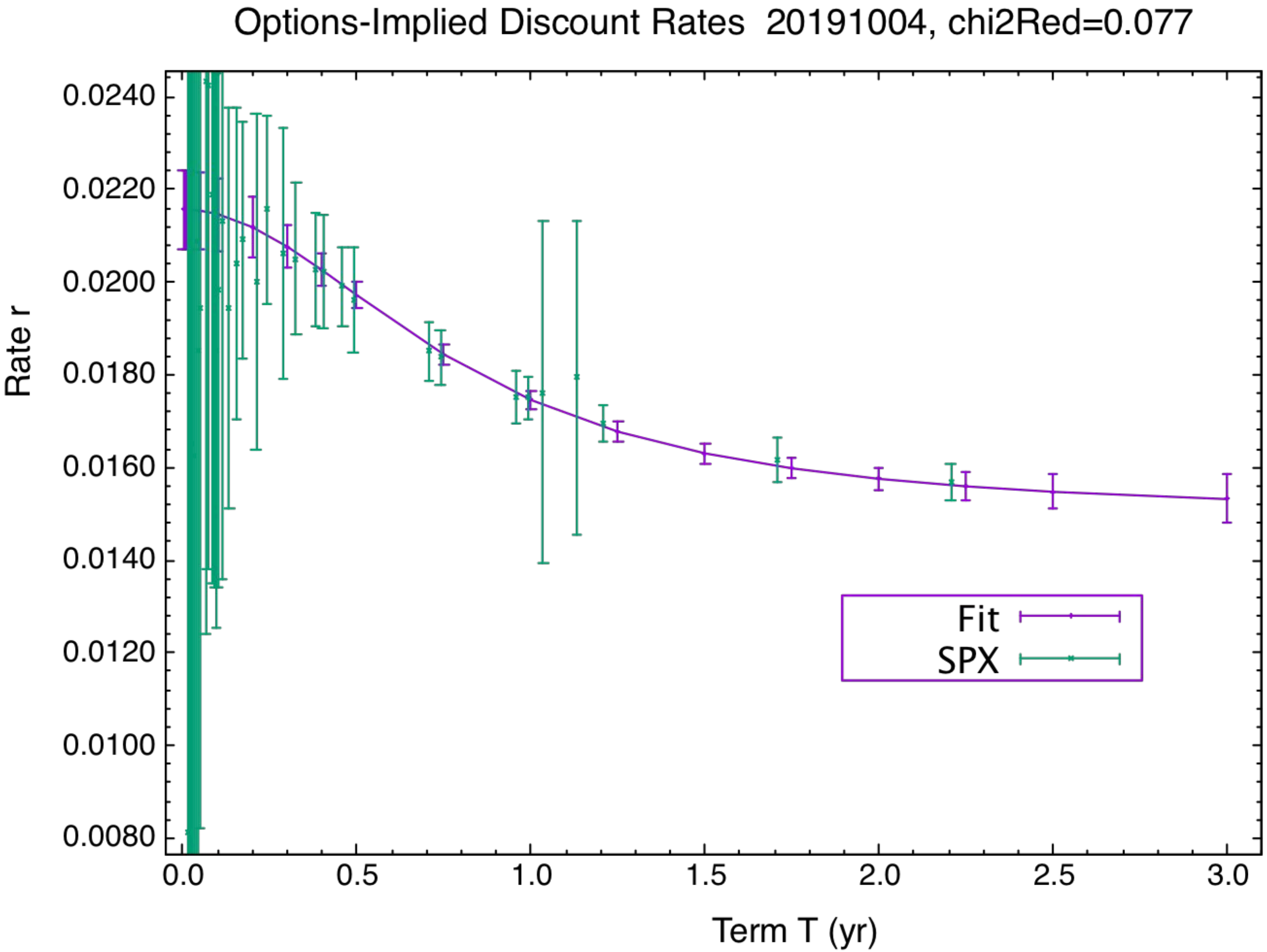
Just one snapshot!

Nelson-Siegel TS fit



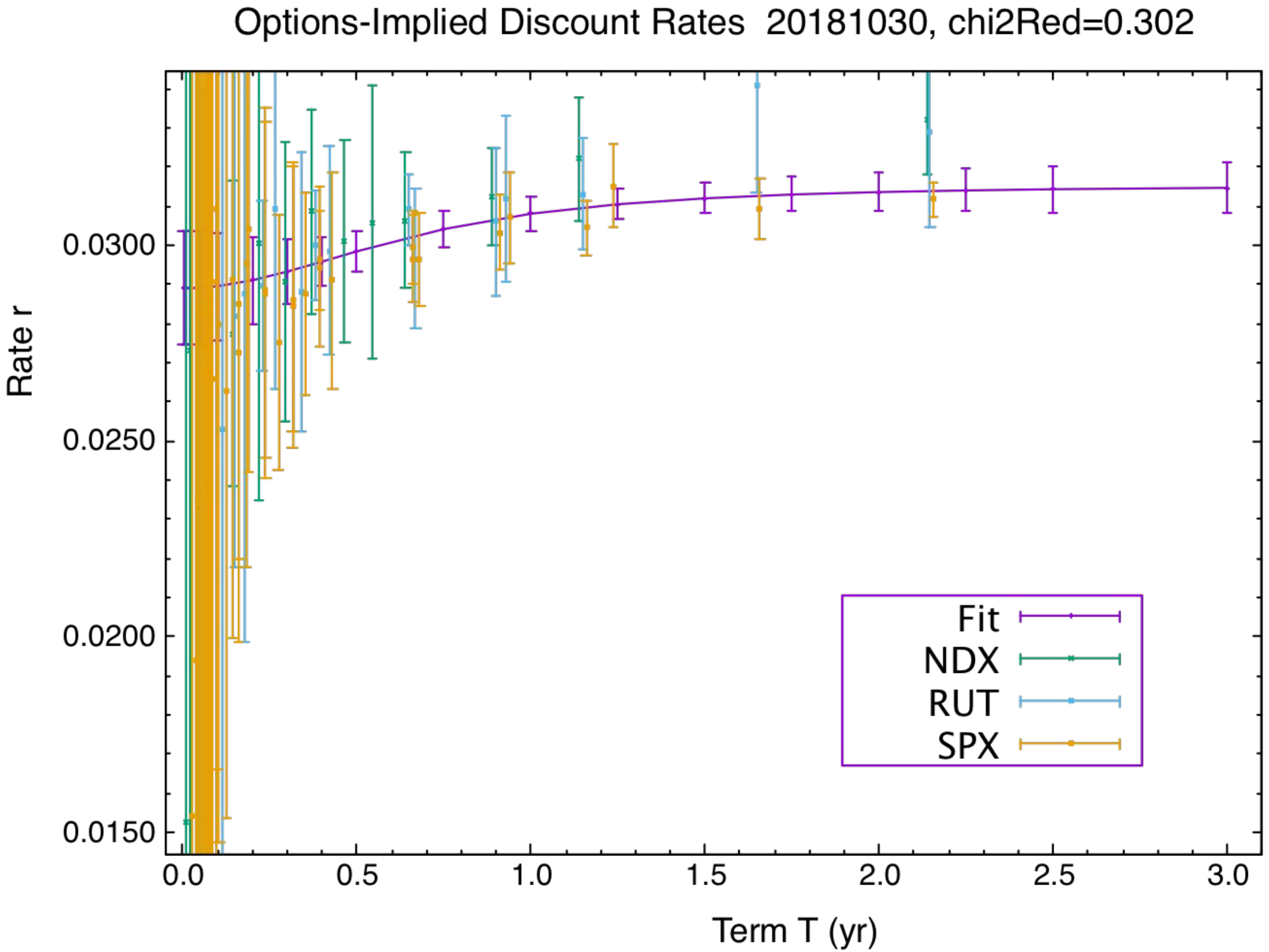
What discount rates should I use?

SPIBOR



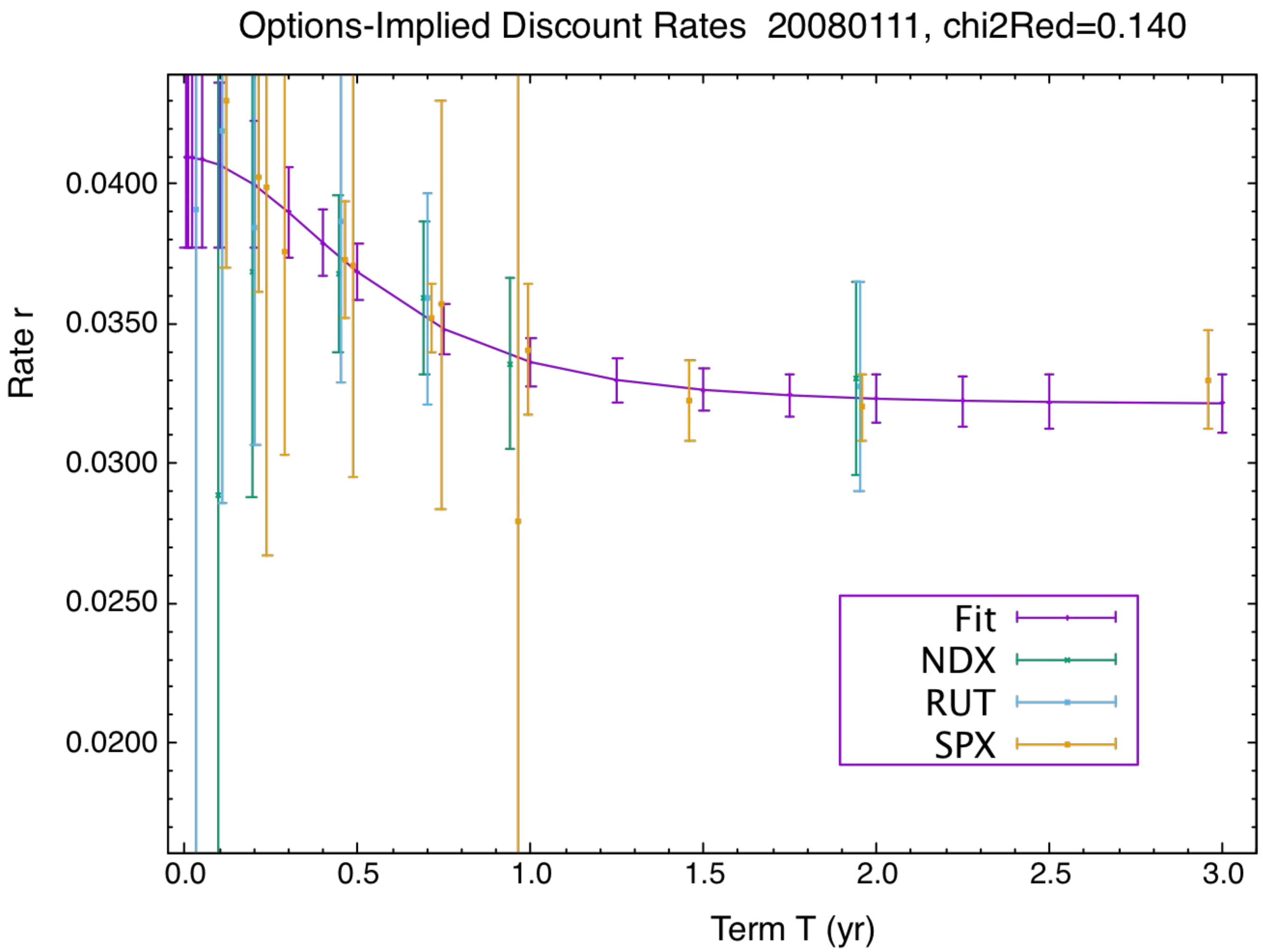
What discount rates should I use?

SPIBOR



What discount rates should I use?

Maybe they are underlier/ sector dependent?



What discount rates should I use in **2008** ??

SPIBOR

Vol Curve/Surface Parametrizations

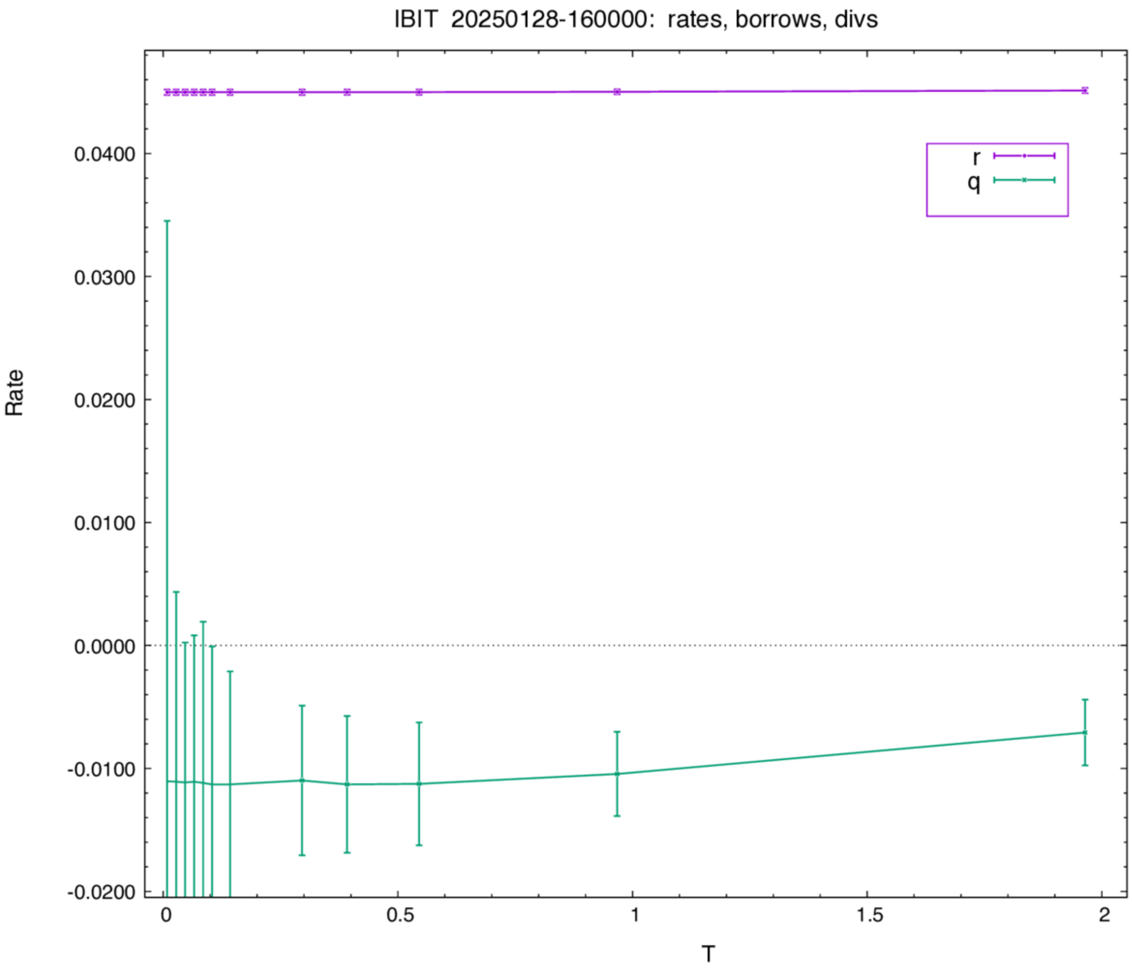
- There are advantages to having a good vol curve parametrization (by strike).
- For current purposes all we have to know is:
  - There are various curves in the public domain, e.g. **S* curves**:
    1. SVI / S5 (5 params per term)
    2. SSVI / S3 (3 params per term)
    3. SABR (3 params per term; name is overloaded: model and curve...)
  - The S* curves do not have much shape flexibility, e.g. they do not allow W-shapes as required around events.
  - Hence there are many proprietary curves out there... We have **C* curves**.

Example: IBIT options

- IBIT = iShares Bitcoin ETF, launched 2024-02-15.
- One of the most successful ETF launches in decades: ~$60bn AUM
- Options started trading 2024-11-19: Very quickly became very liquid.
- We will show:
    - Implied borrow costs q(T).
    - Vol fits: SVI vs C9W, with metrics like chi2Reds, avE5 (avgErrors5)
    - Theoretical prices (“theos”) vs market prices
    - No arbitrage: Densities, total variances

How to imply the borrow cost q(T)?

- For European-style options we could use PCP to imply the forward for each expiry, and then q(T) from the forwards.
- For American-style options PCP does not hold. Call and/or put prices could have an early-exercise premium (EEP), even around ATM.
- Time-honored tradition is to use **“American PCP”** as:
  - For strikes K around ATM demand: volPut(K) = volCall(K)
- This is not really true! One can check this in any real model, like LV, SLV.
- It holds “well enough” if EEP are small-ish compared to spreads for some strikes around ATM — OR if this is what “the market” does!
- For now, imply the borrow such that American PCP holds.
  - If pricing with rate term-structure, bootstrapping from small to large T is needed.



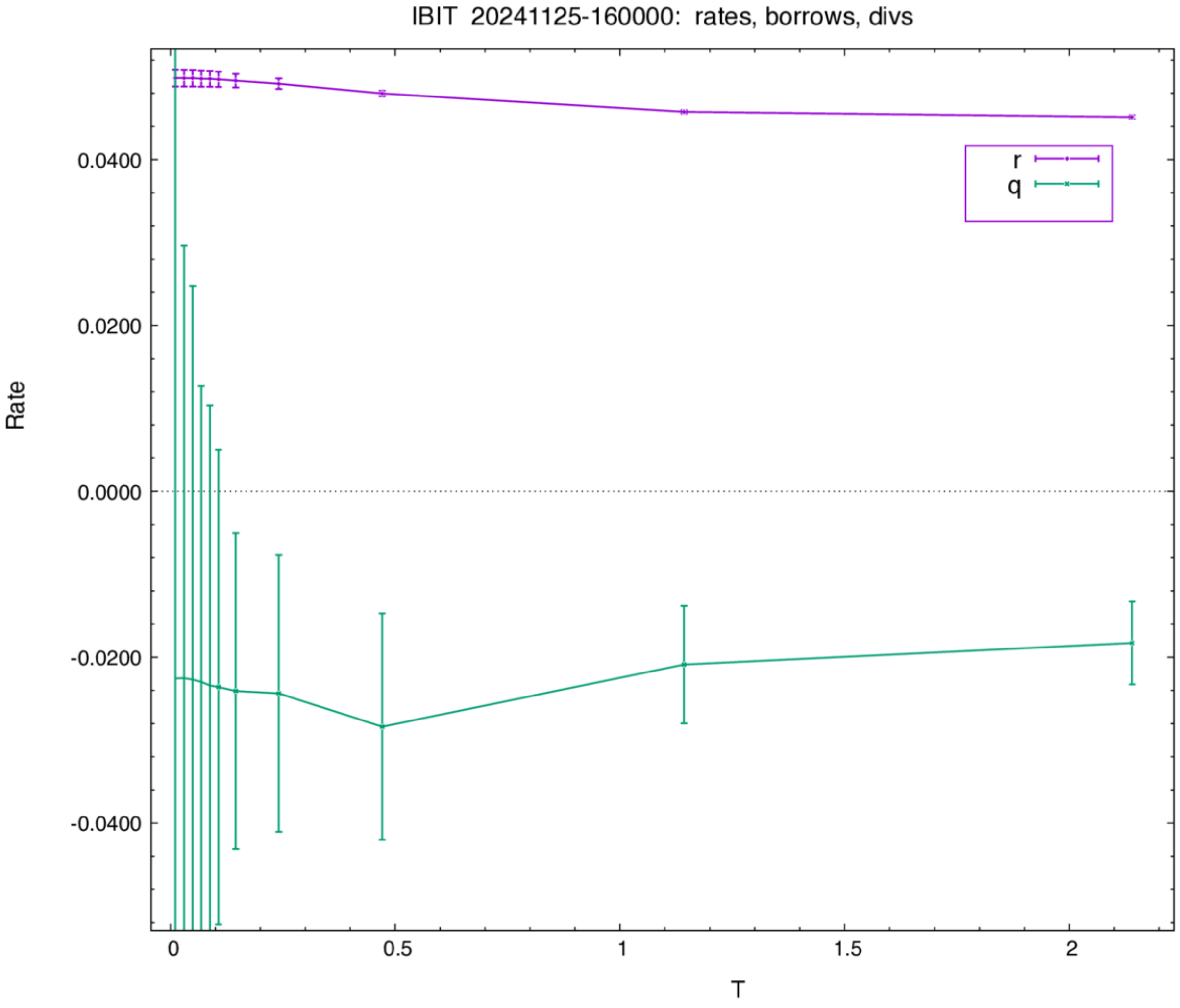
IBIT 2025-01-28

Implied borrow cost by term q(T)

The actual borrow cost is +1.2%

Massive demand for upside leverage leads to much smaller implied q(T) !

We also implied the spot here.



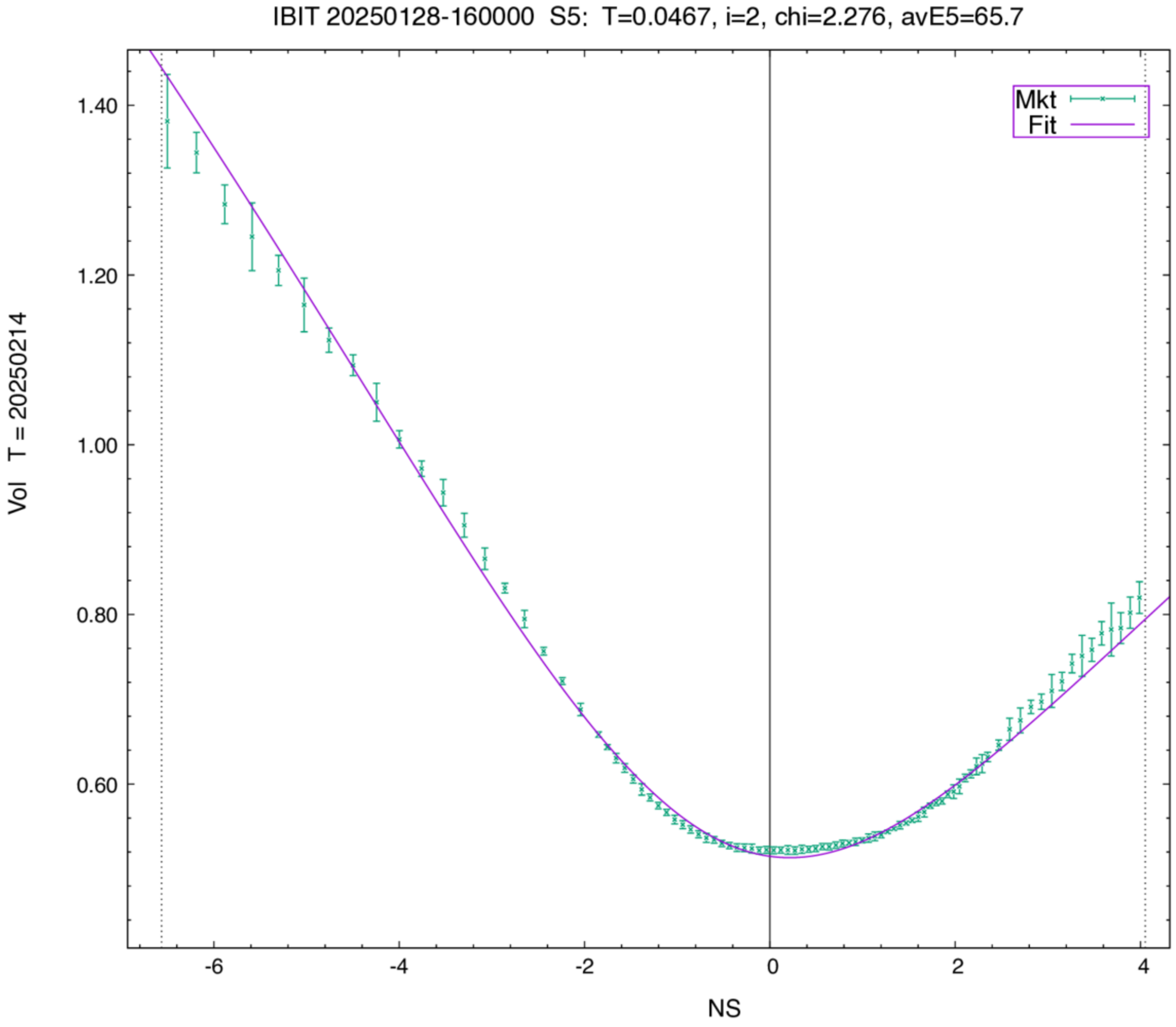
IBIT 2024-11-25

A few days after options trading started

Implied borrow cost by term q(T)

Massive demand for upside leverage leads to **much** smaller implied q(T) !

We also implied the spot here.

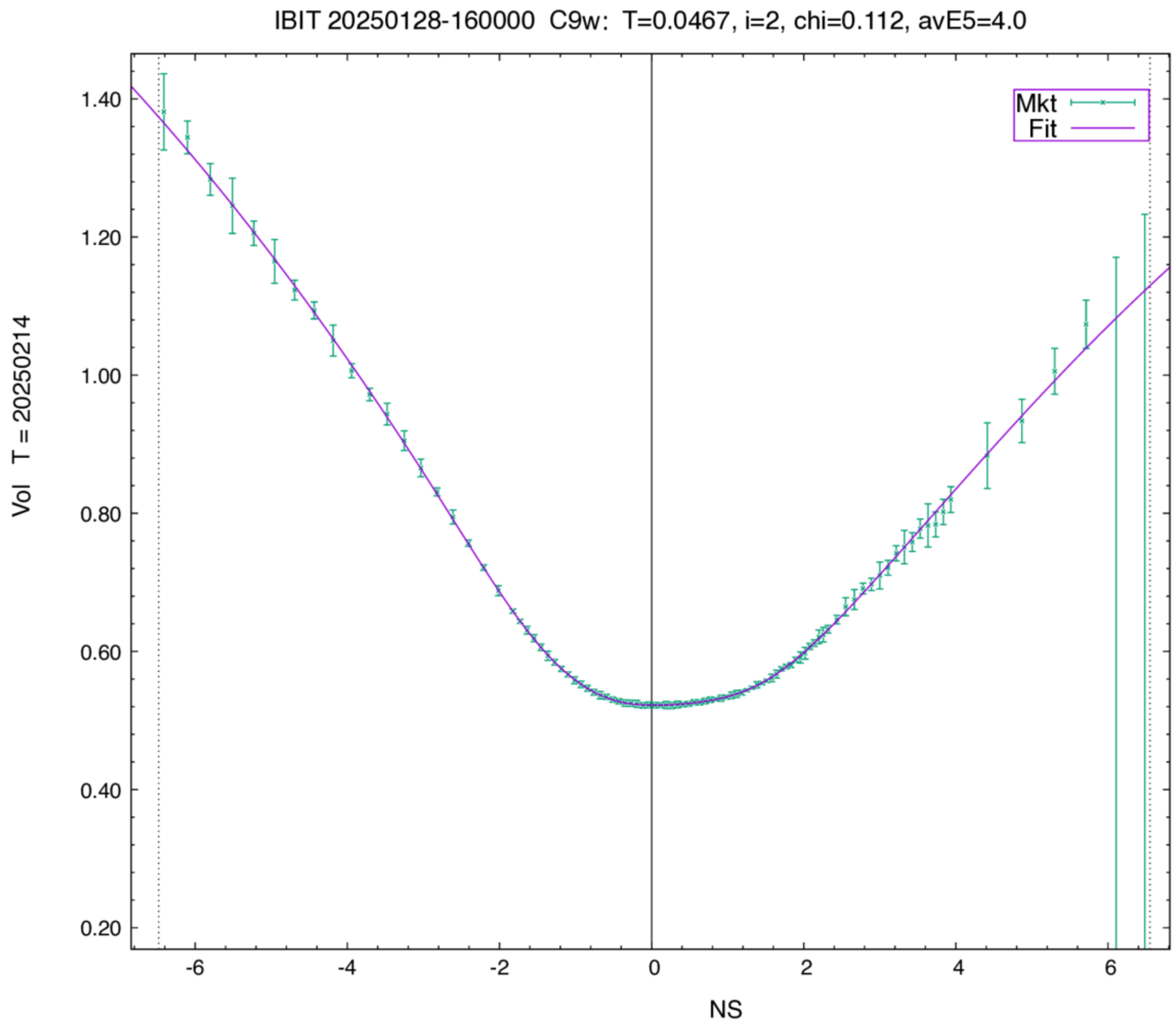


IBIT 20250128 16:00

S5, T = 2w, NS space

S5 = **SVI**

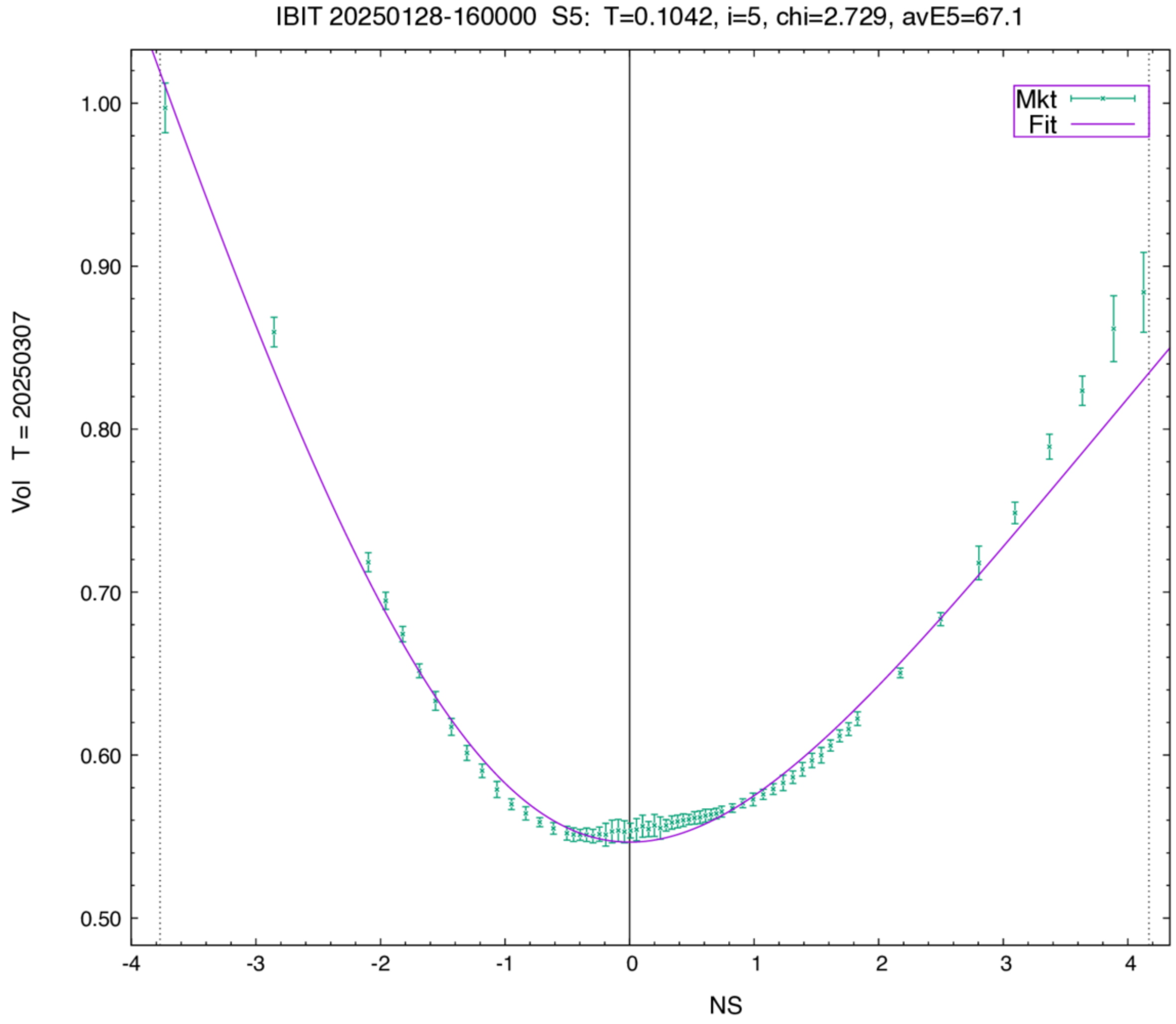
$$z := \mathrm{NS} := \frac{\ln(K/F)}{\sigma_0\sqrt{T}}$$



IBIT 20250128 16:00

C9w, T = 2w, NS space

Fit is 15-20x better than SVI



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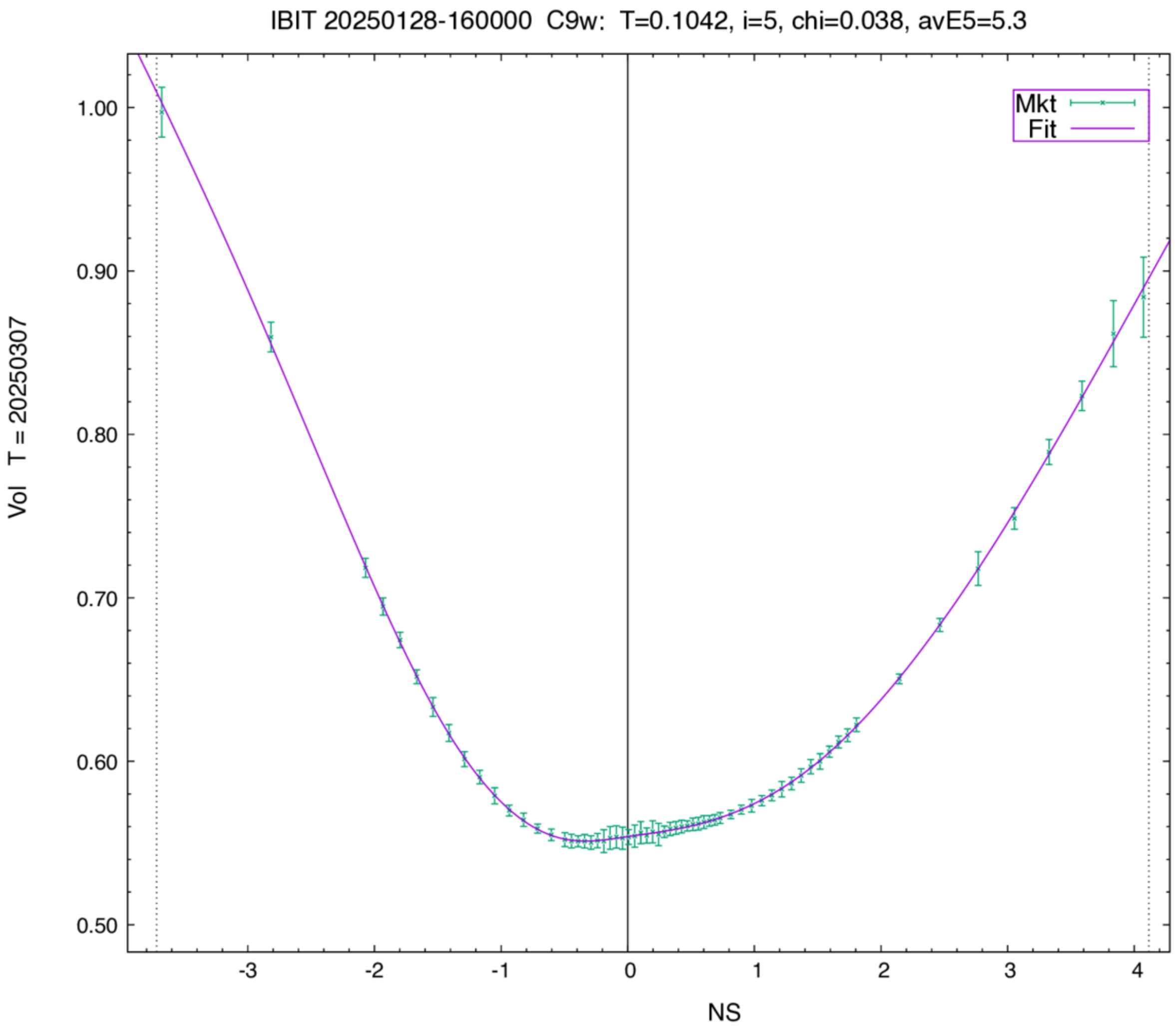
S5 T = 5w, NS space

$$z := \mathrm{NS} := \frac{\ln(K/F)}{\sigma_0\sqrt{T}}$$



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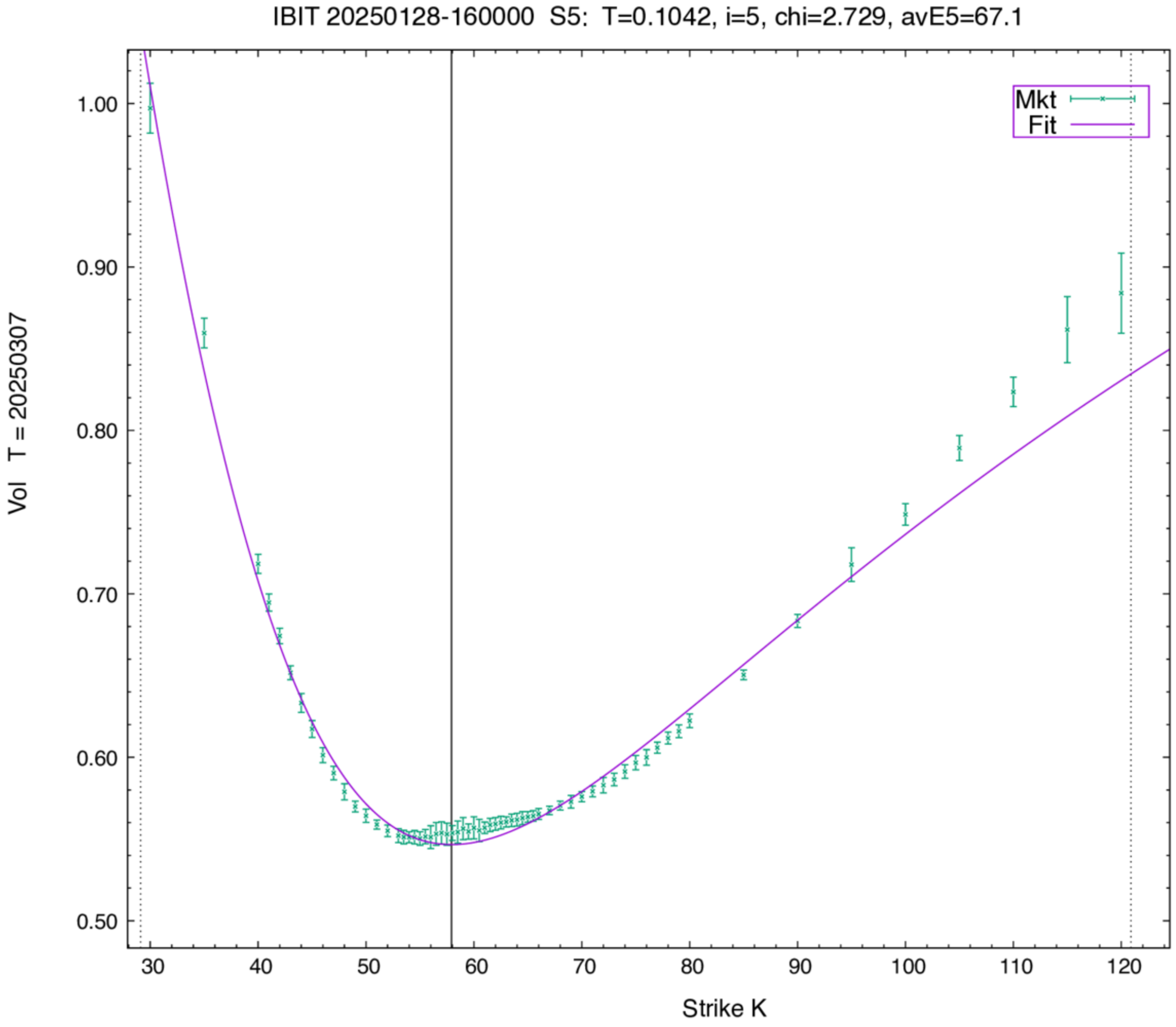
C9w, T = 5w, NS space

Fit is 13-70x better than SVI



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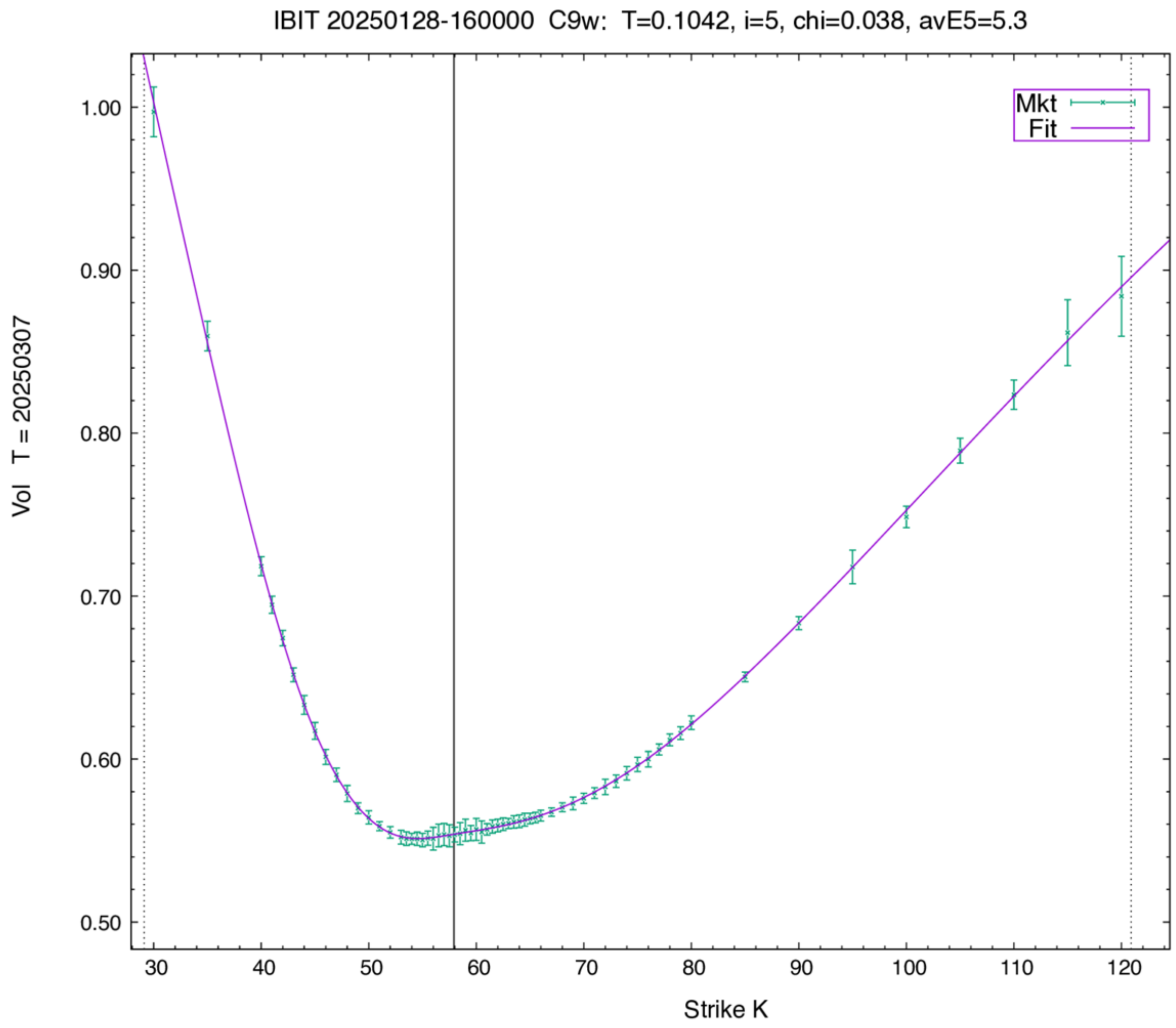
S5 T = 5w, K space



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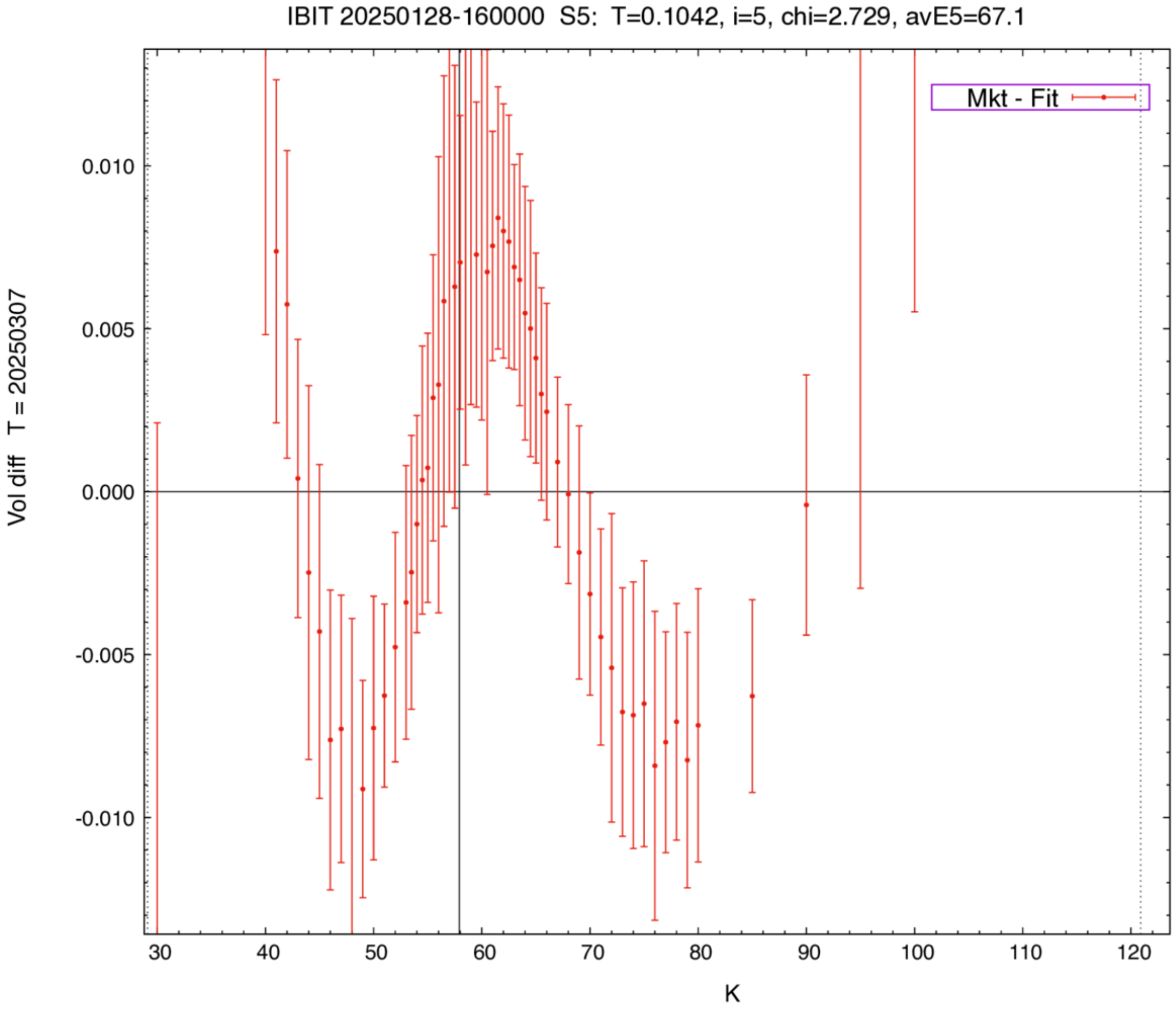

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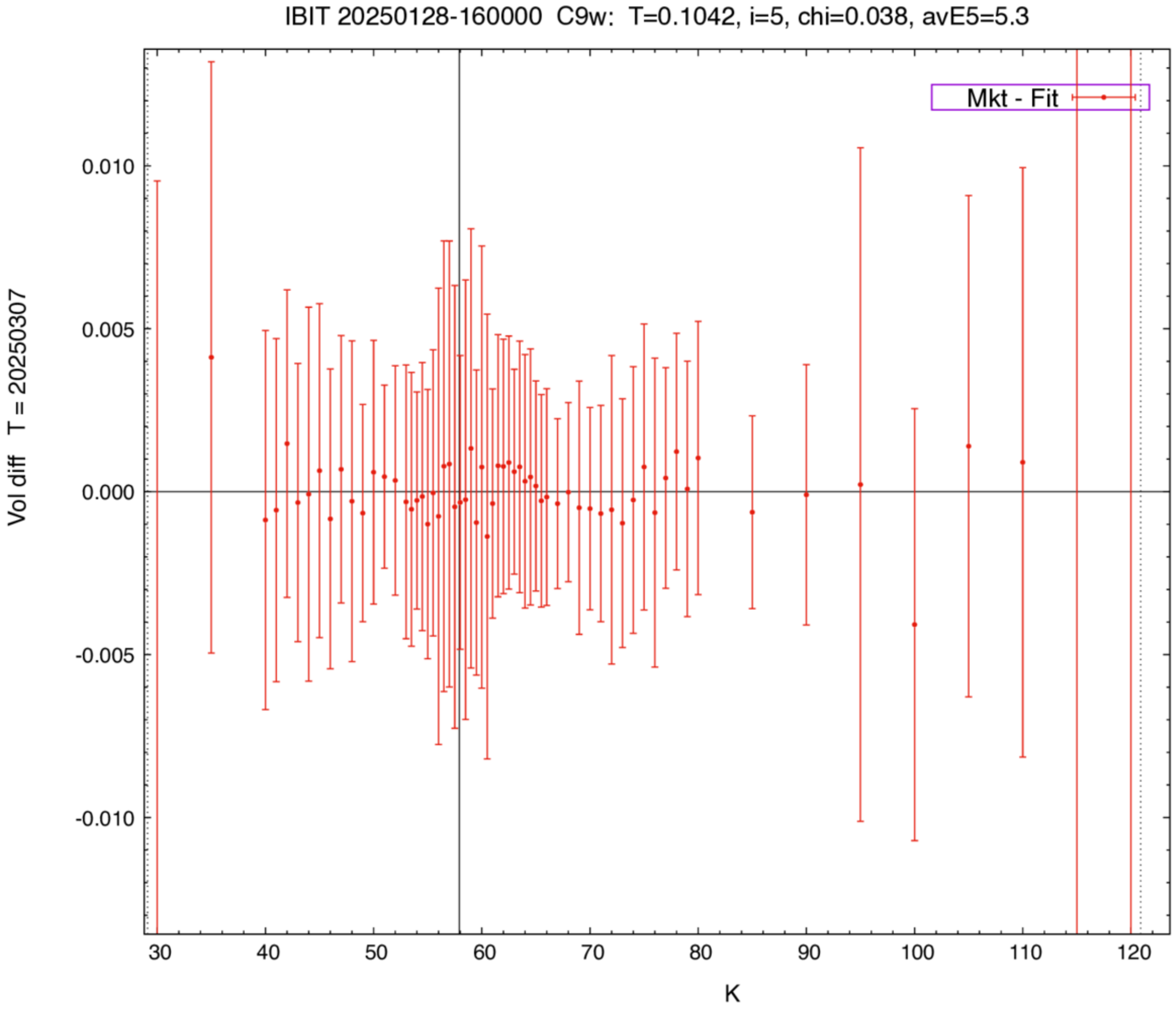
S5 T = 5w, K space

"Vol diff" plot



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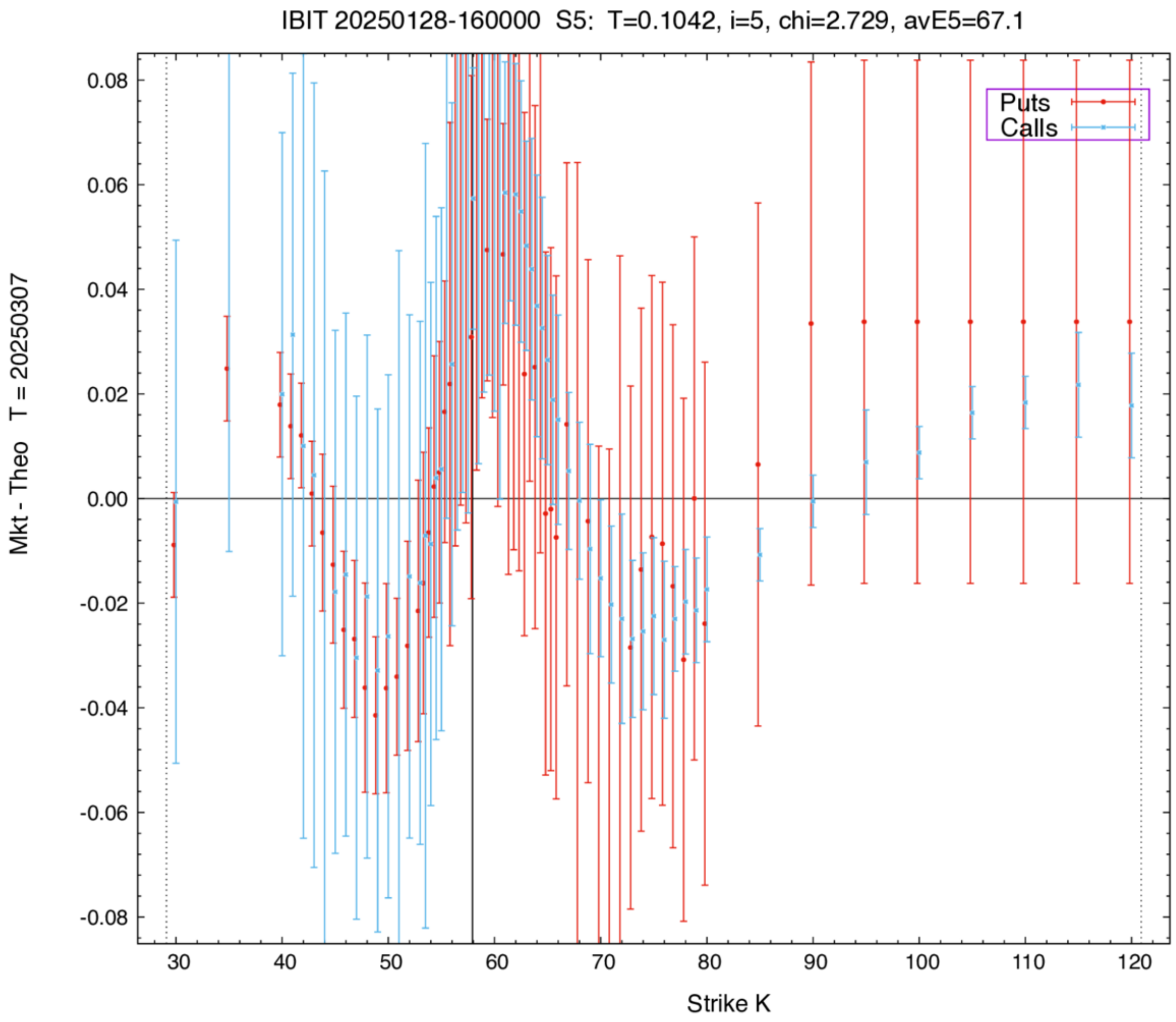

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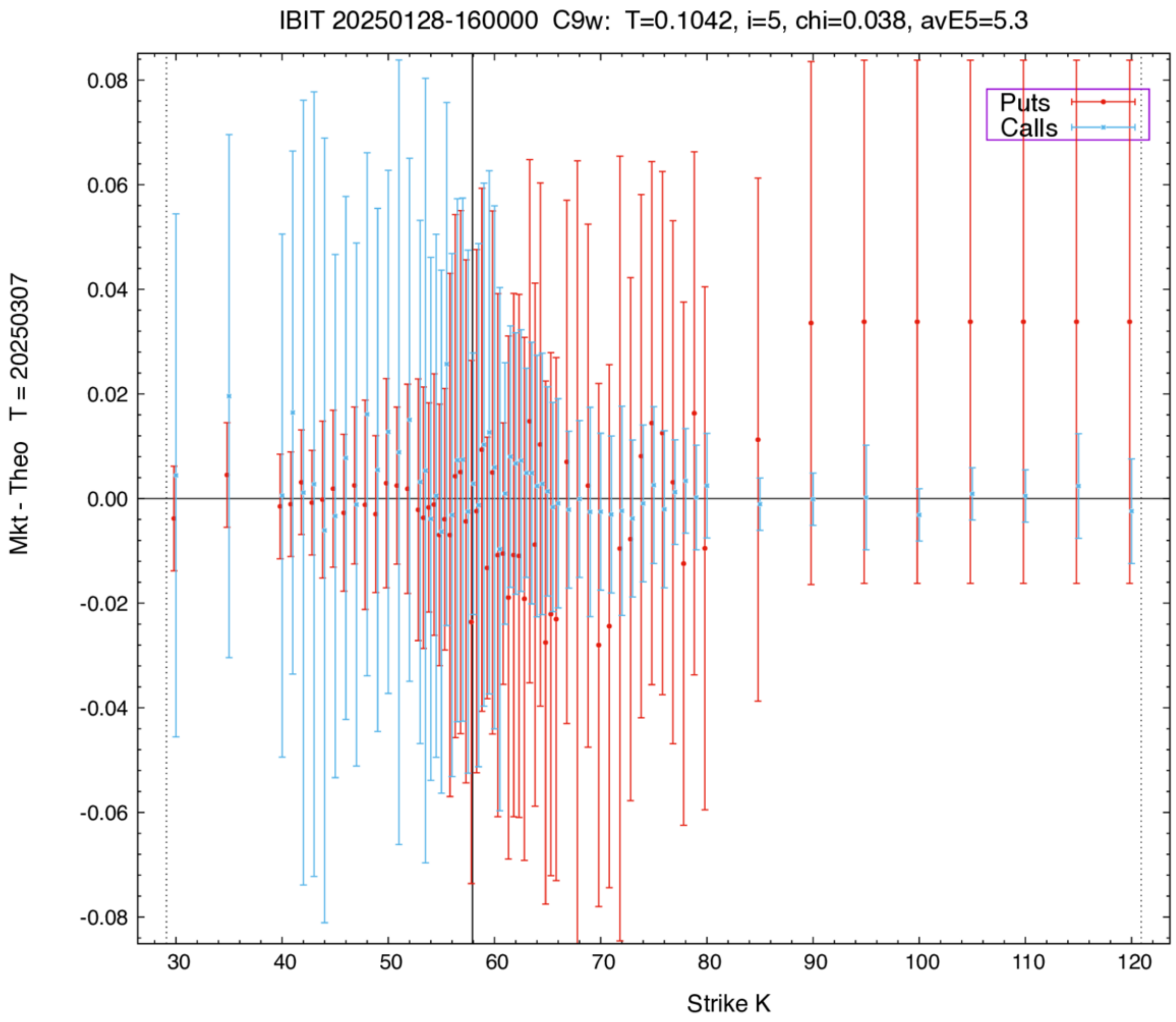
S5 T = 5w, K space

"Price diff" plot: The ultimate test!



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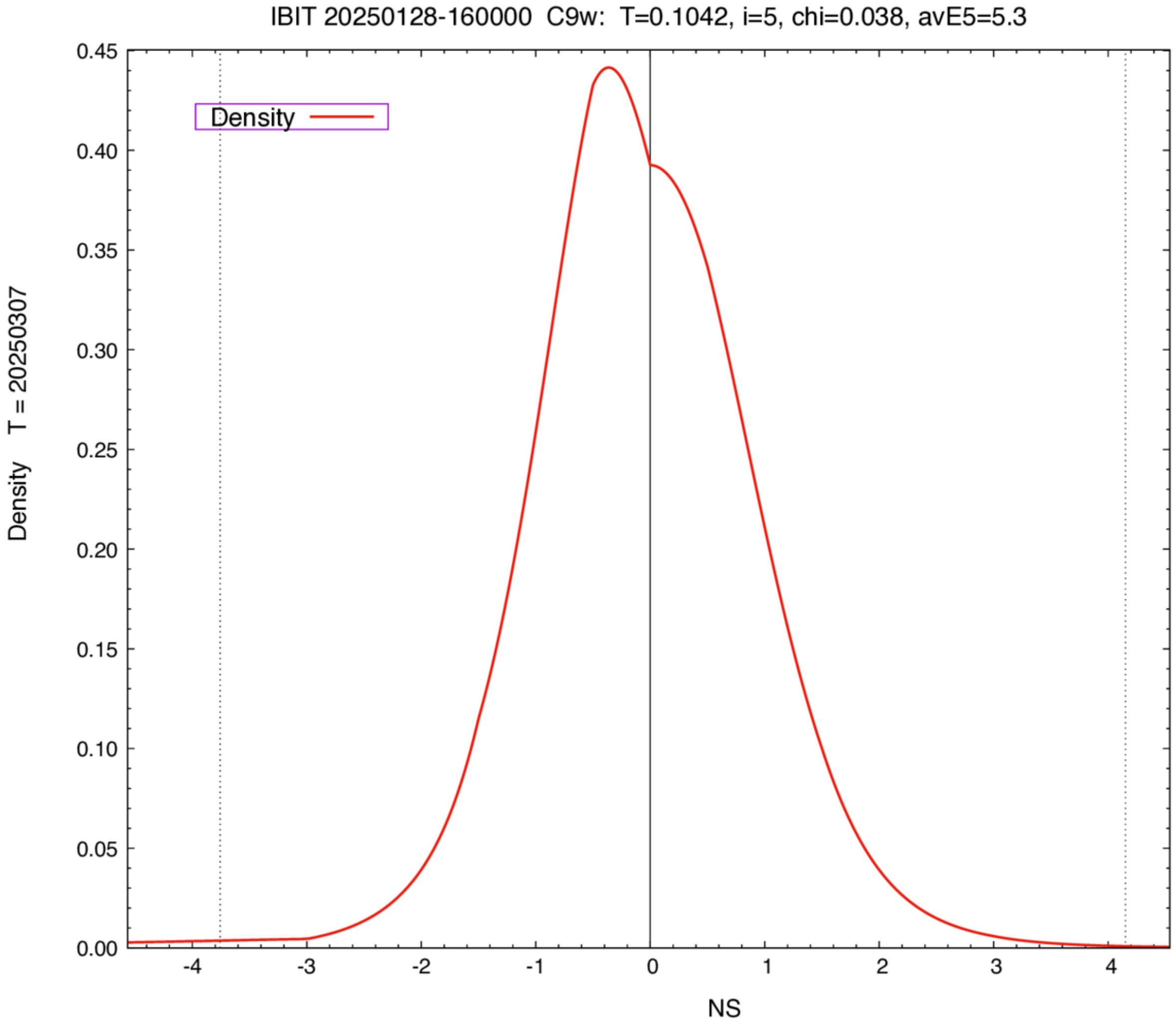
C9w, T = 5w, K space

“Price diff” plot: The ultimate test!

Fit is 13-70x better than SVI



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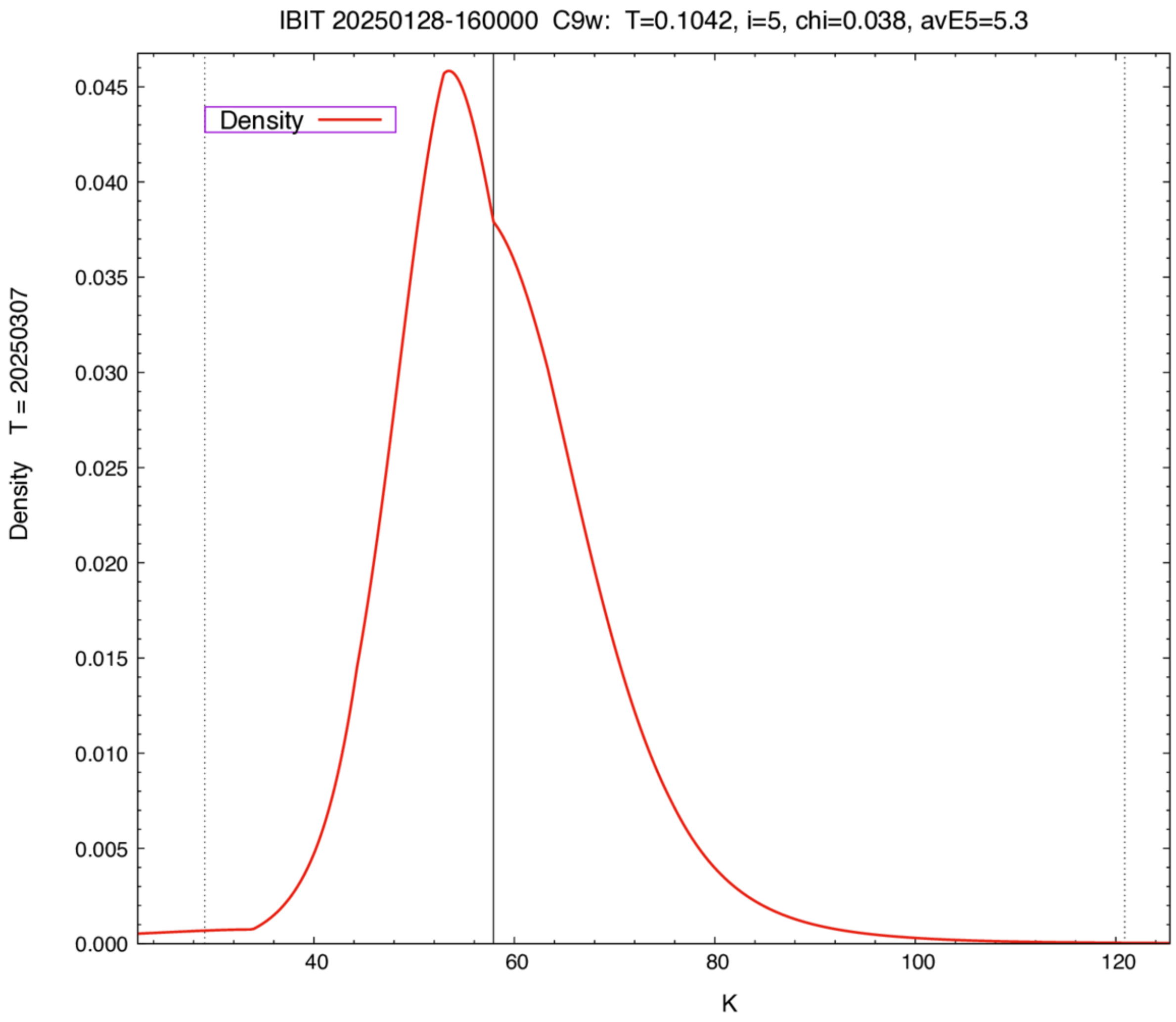
C9w, T = 5w, NS space

Implied density

A pretty fat put wing…



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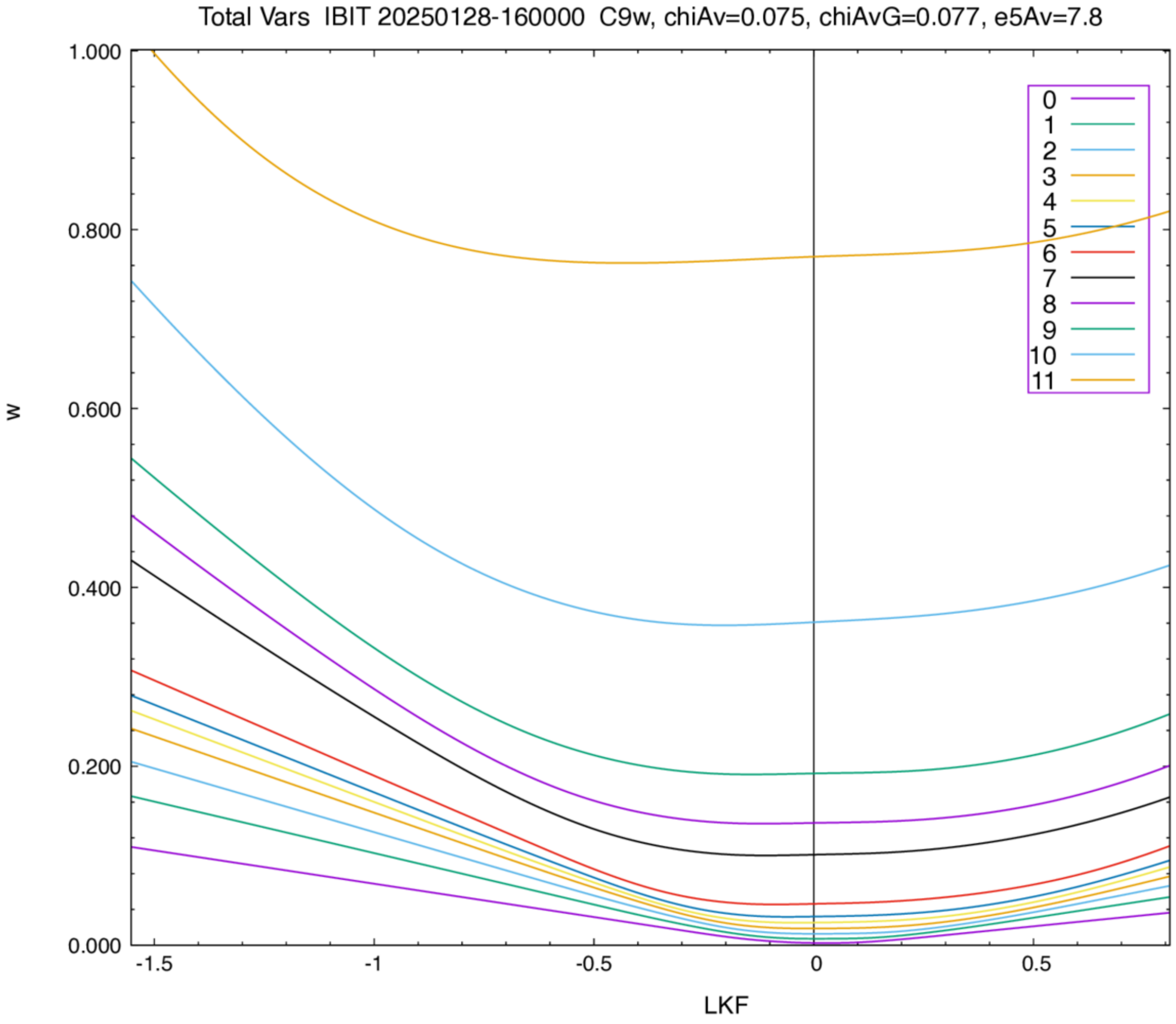
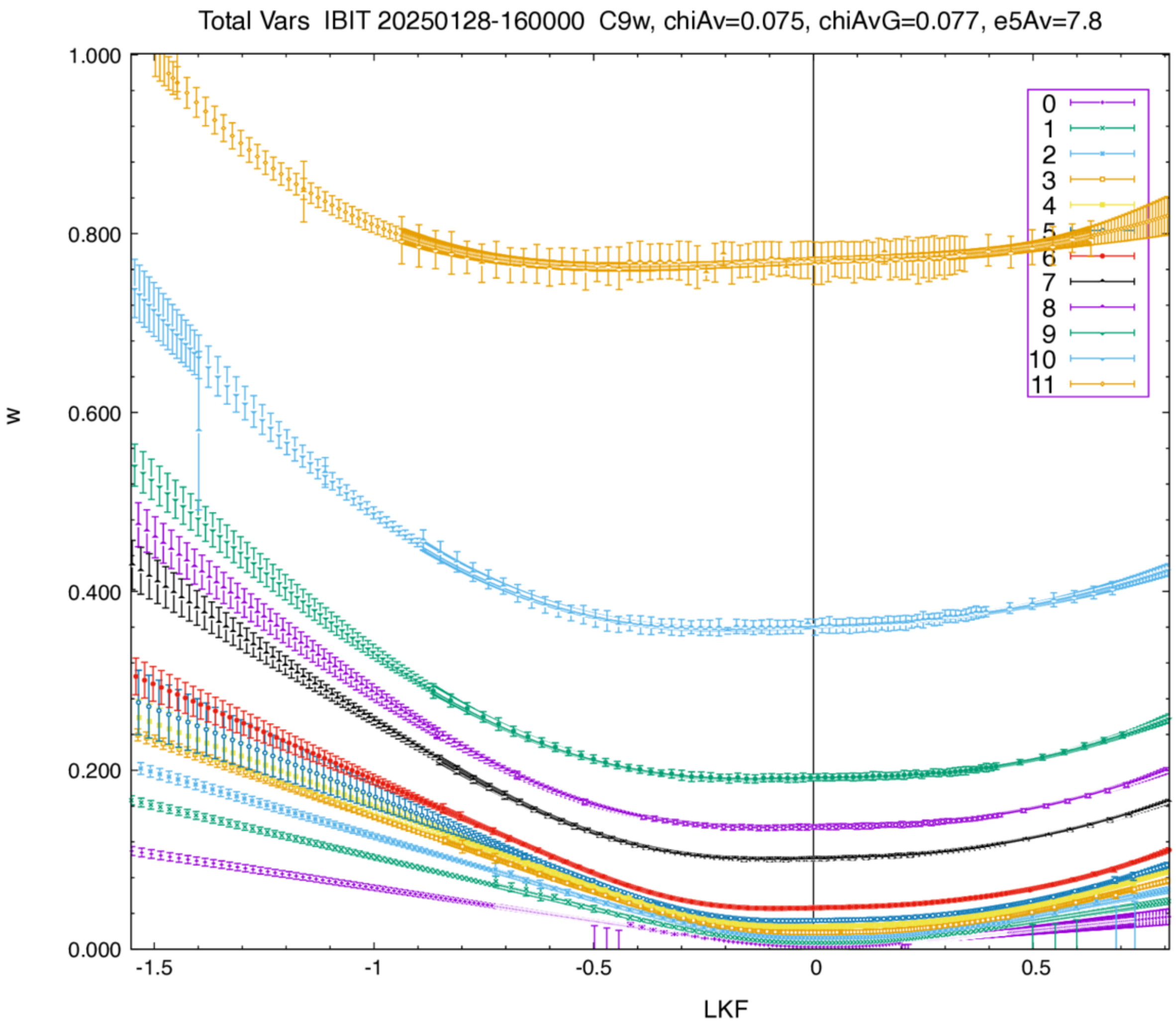


IBIT 20250128 16:00

Total Variance plot

No calendar arbitrage!

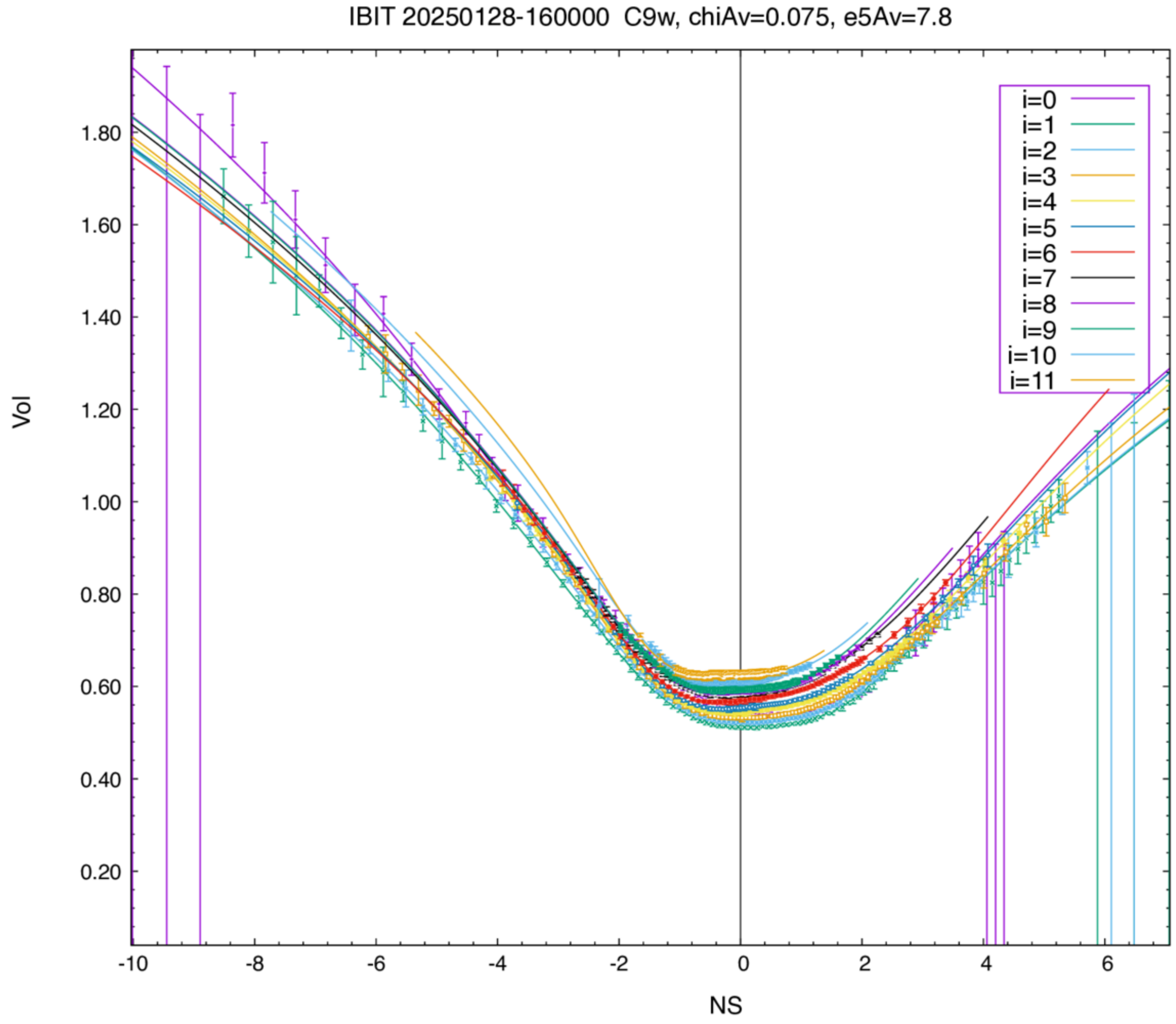
$$w(T,K) := \sigma(T,K)^2\ T$$



IBIT 20250128 16:00

Total Variance plot

With error bars!



IBIT 20250128 16:00

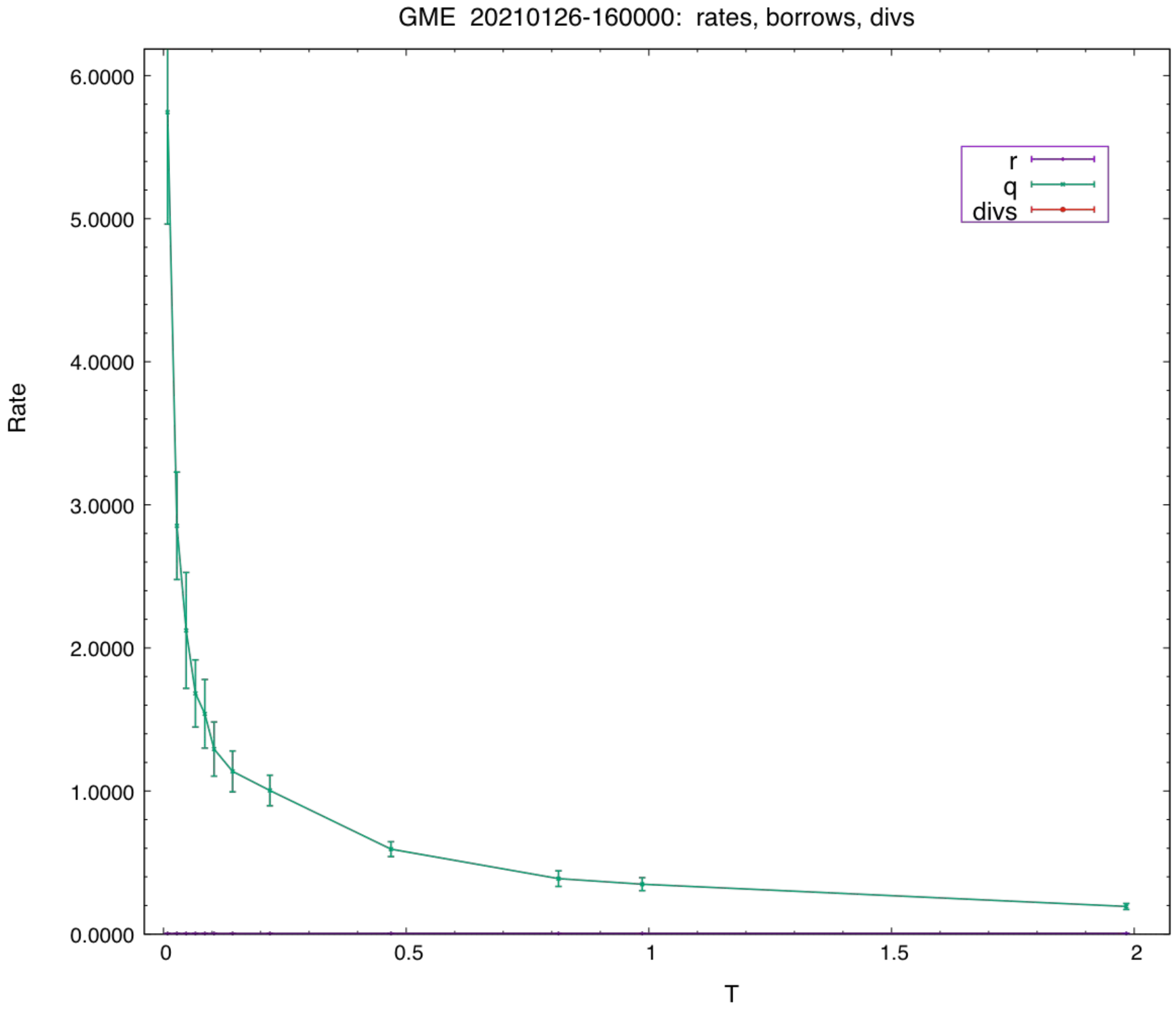
All vols by NS

IBIT options: Conclusions

- Implied borrows can be very different from actual (overnight) borrow fees charged by agent banks/prime brokers.
- As an options market becomes liquid, SVI will very quickly not be flexible enough to match the market in a bias-free manner (Klassen’s law).

Gamestop during the 2021 short squeeze

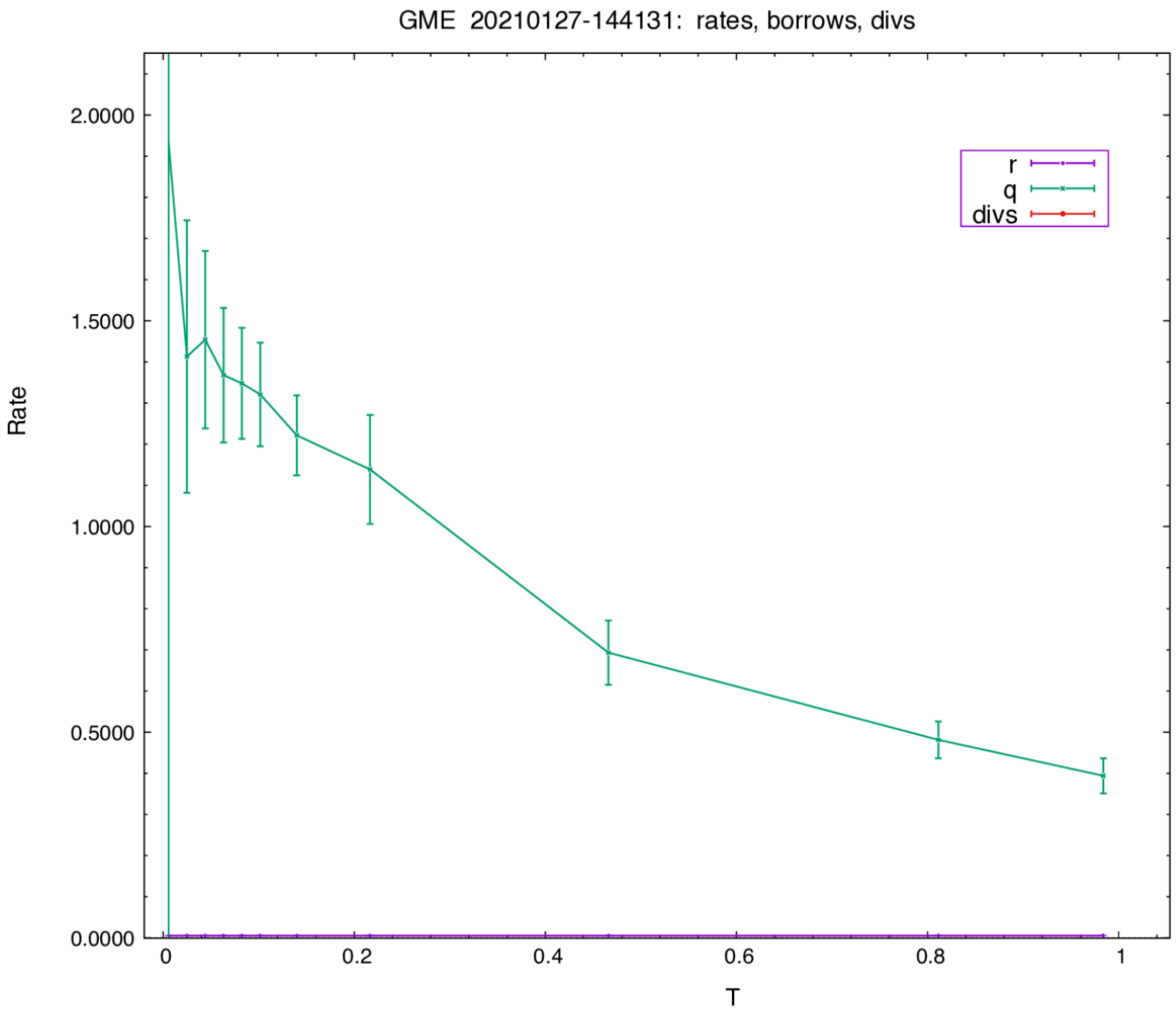
- GME was one of the most prominent meme stocks during the 2021 craziness.
- It made Roaring Kitty & reddit/WallStreetBets famous, killed Melvin Capital, lead to trading restrictions on Robinhood, numerous lawsuits, and conspiracy theories — and maybe(?) some better understanding by the retail public of how margin accounts and short selling works…
- Both stock and options trading exploded — supposedly more premium traded in GME than in SPX options for a couple of days!
- See wikipedia or my 2021 LinkedIn post for details.



Gamestop 2021-01-26

Implied Borrow TS: q(T)

Up to 600% !

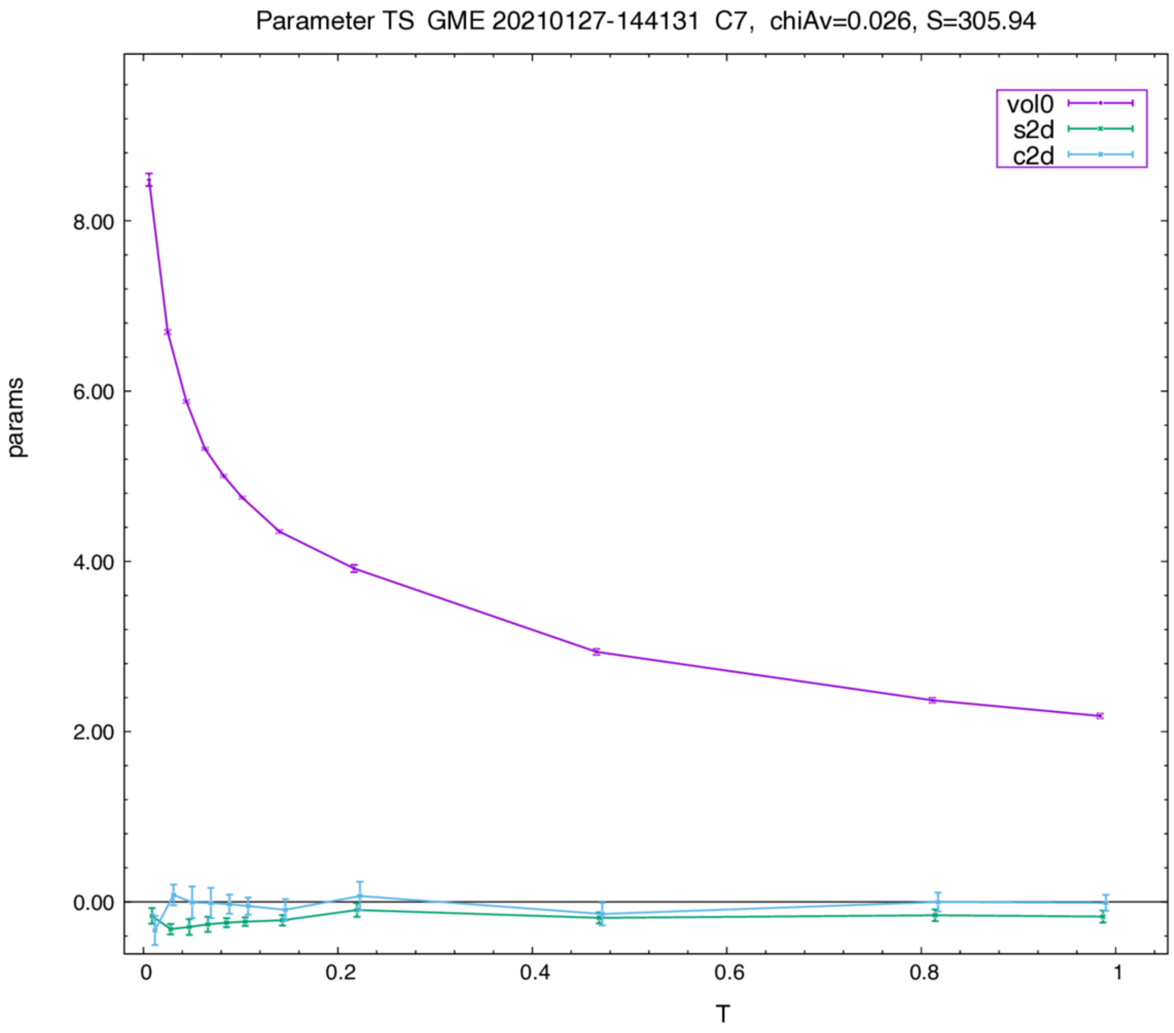


Gamestop 2021-01-27

The day it more than doubled…

Implied Borrow TS: q(T)

Up to 180% !

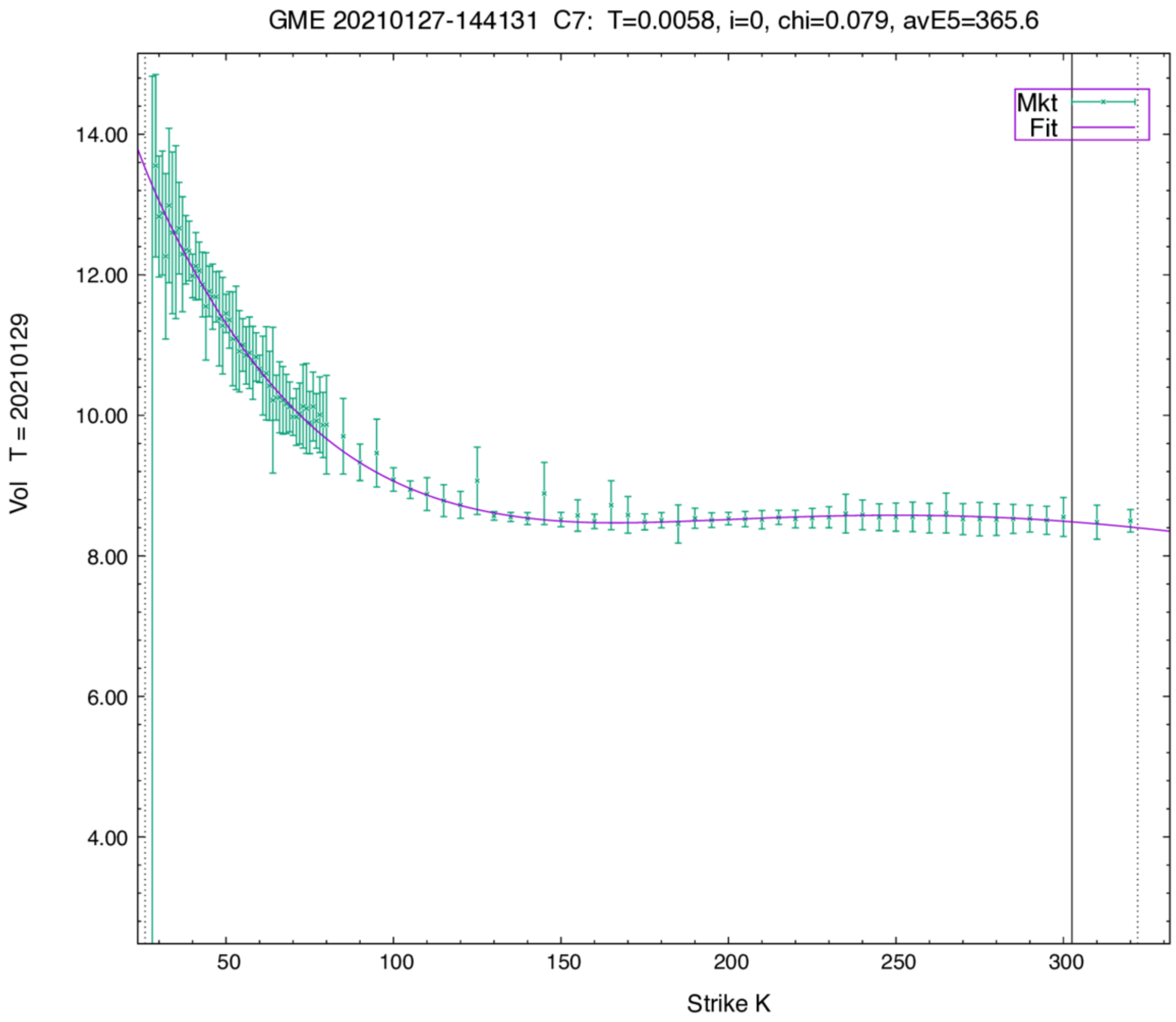


Gamestop 2021-01-27

The day it more than doubled…

Implied Parameter TS

ATM vol above 800% !

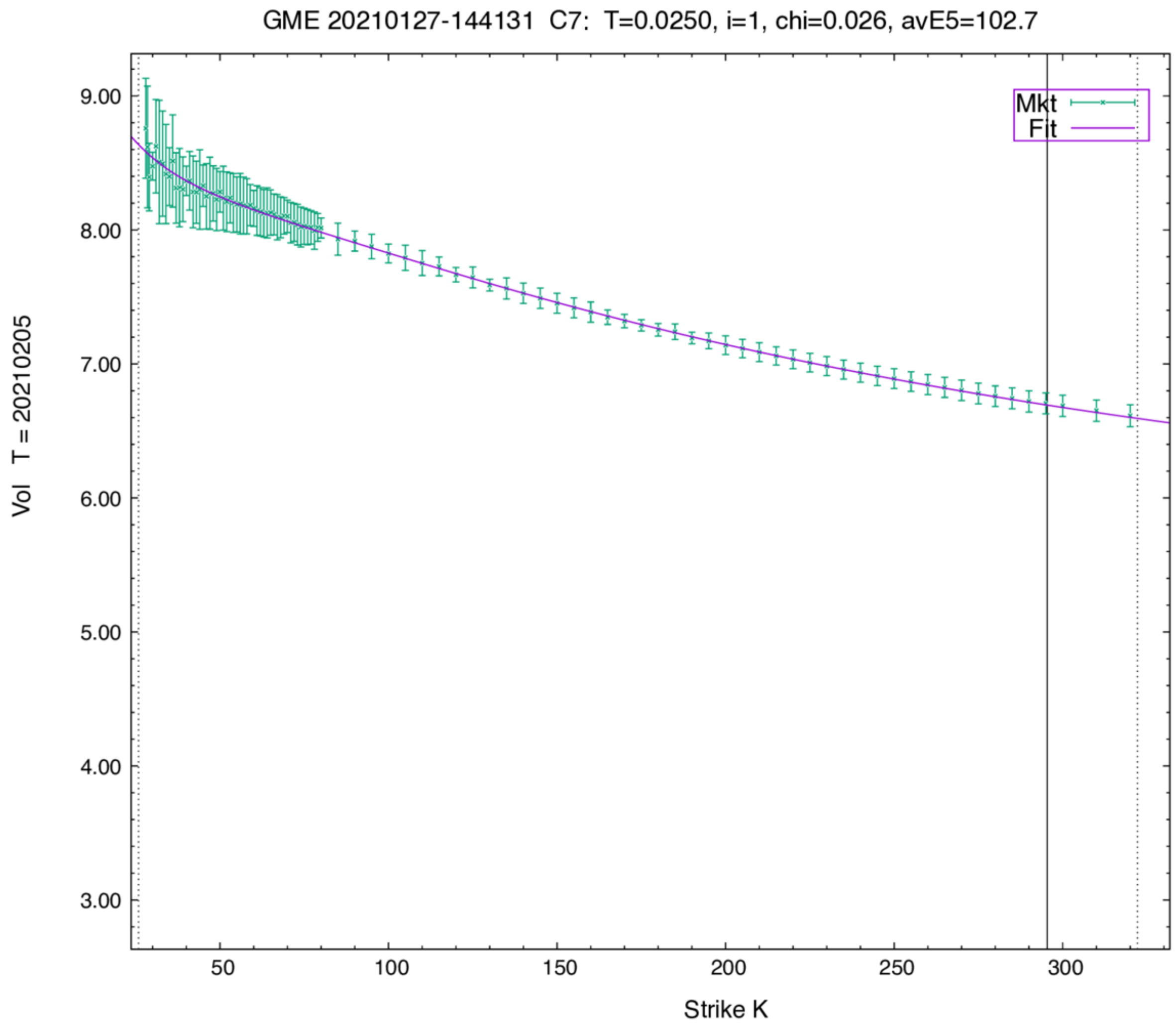


Gamestop 2021-01-27

The day it more than doubled…

Implied vol of 1st Term

It almost ran out of strikes on call side…

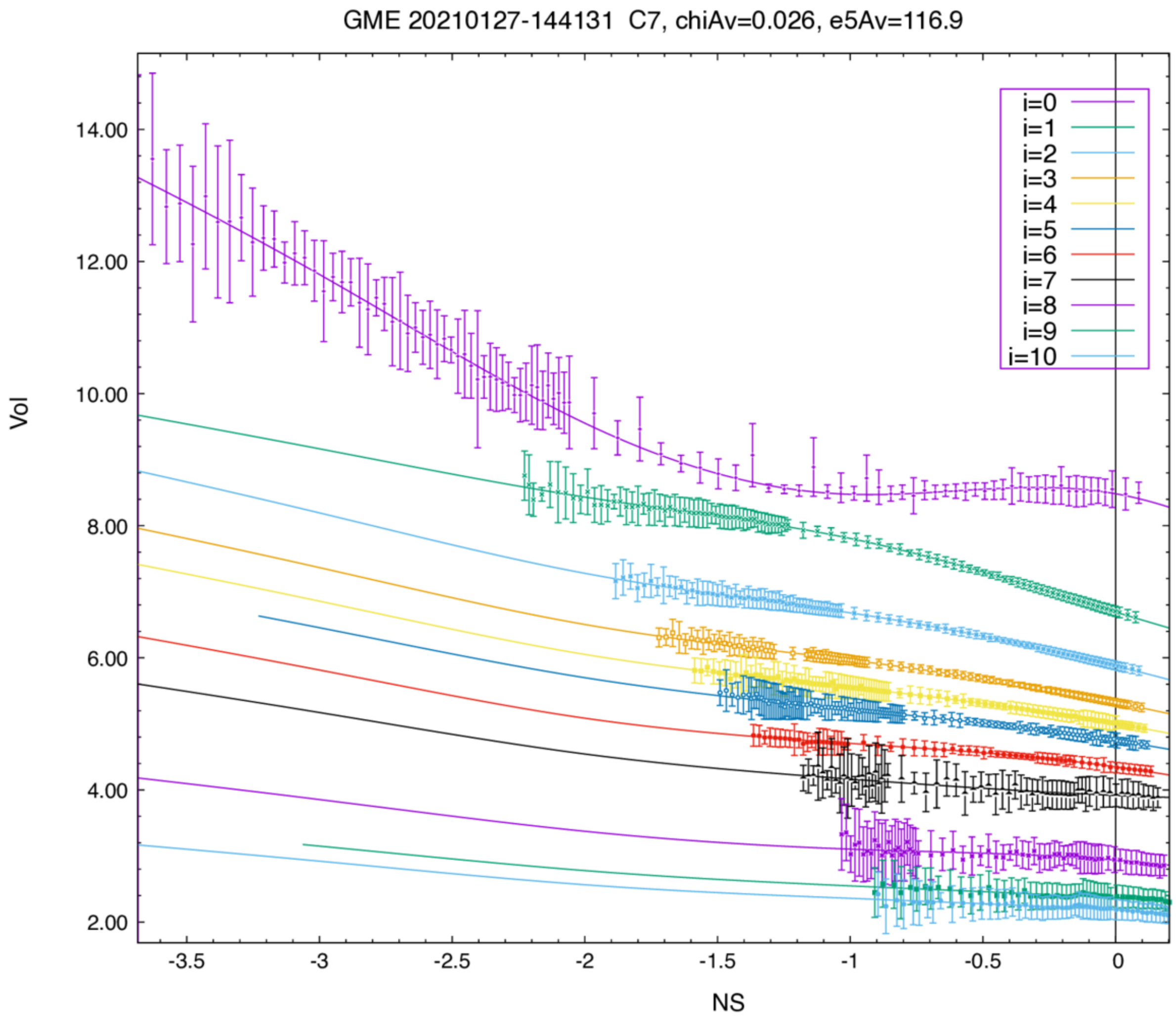


Gamestop 2021-01-27

The day it more than doubled...

Implied vol of 2nd Term

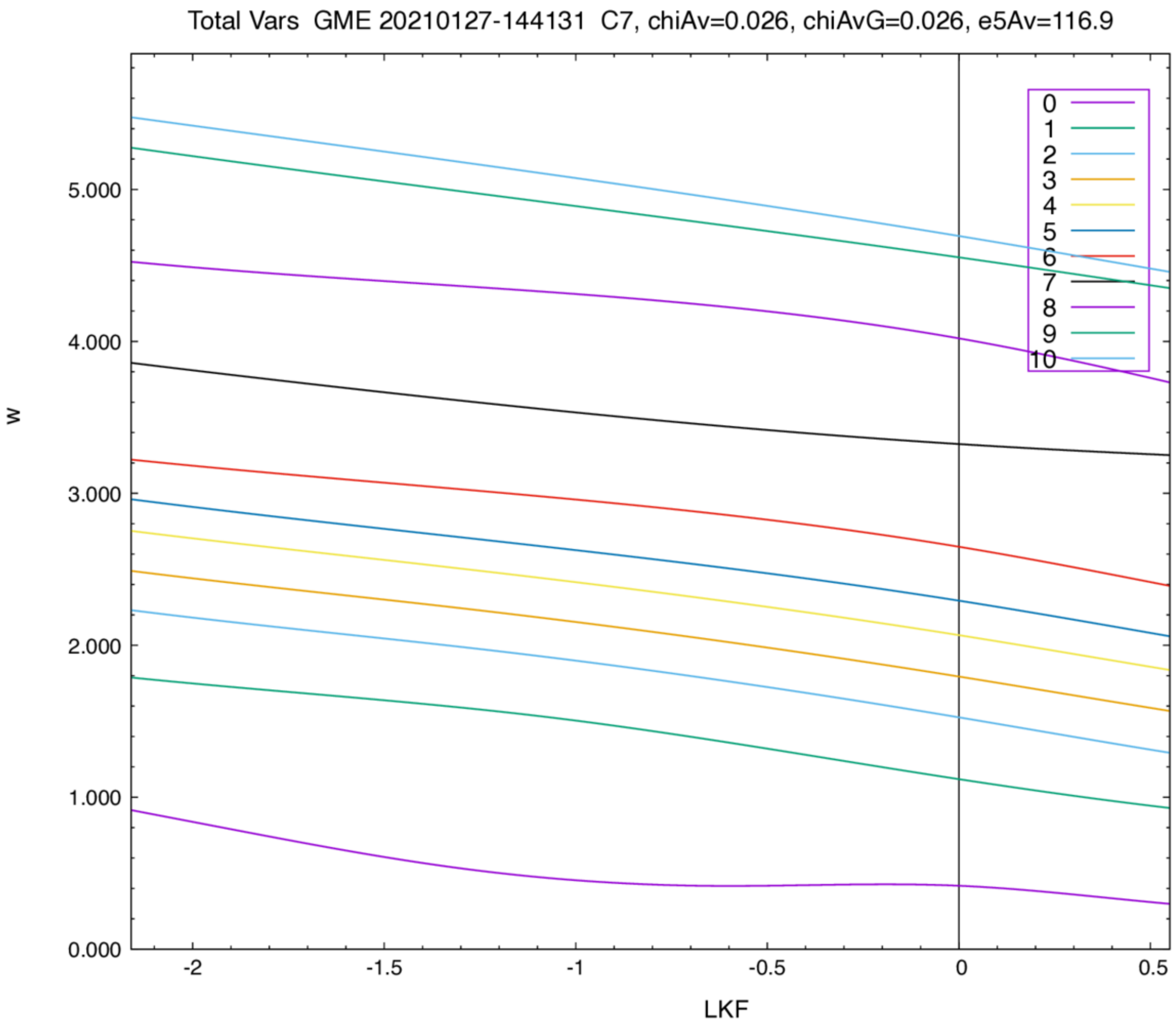
It almost ran out of strikes on call side...



Gamestop 2021-01-27

The day it more than doubled…

Implied vol of all terms by NS

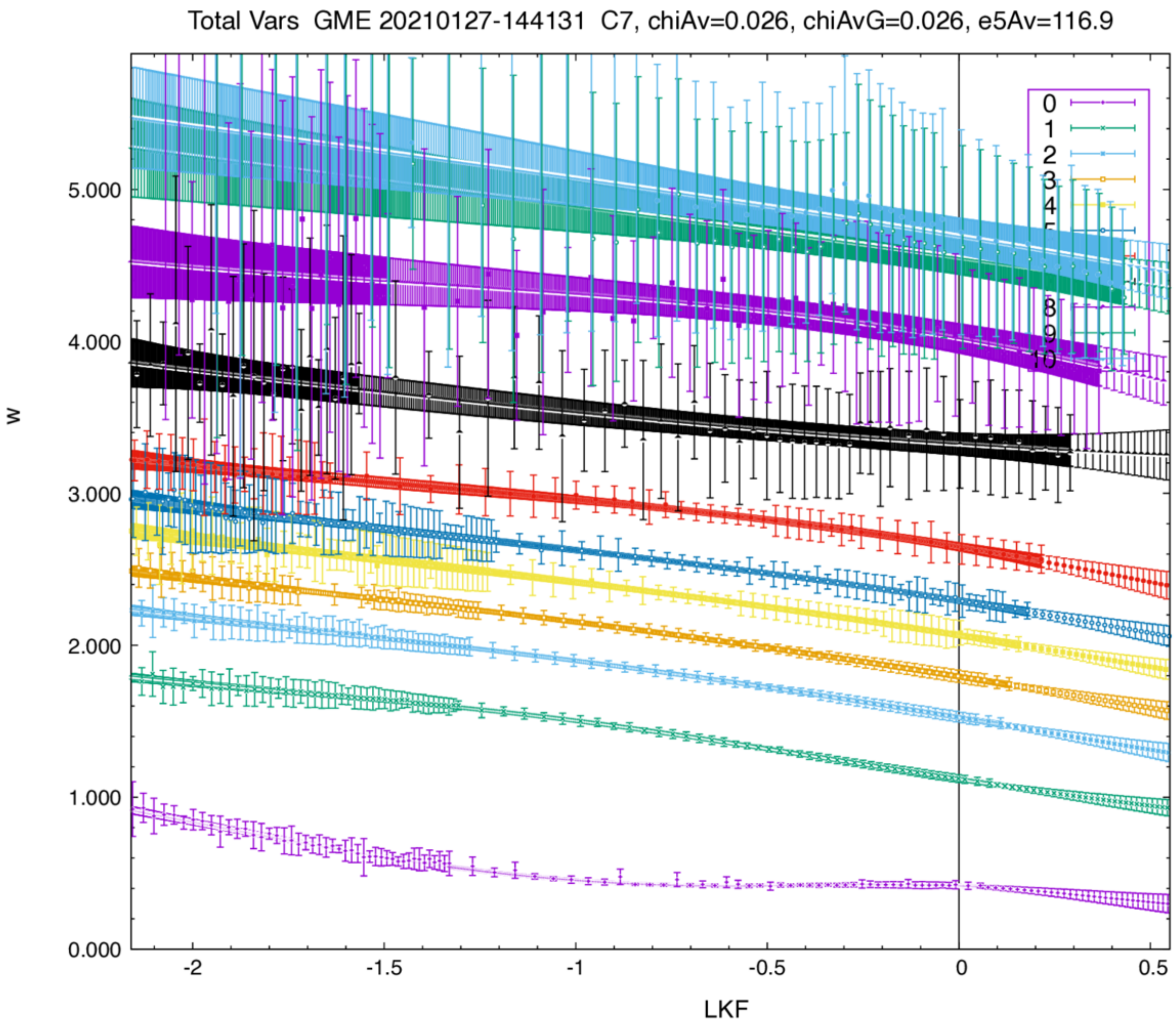


Gamestop 2021-01-27

The day it more than doubled...

Total Variance plot

No calendar arb



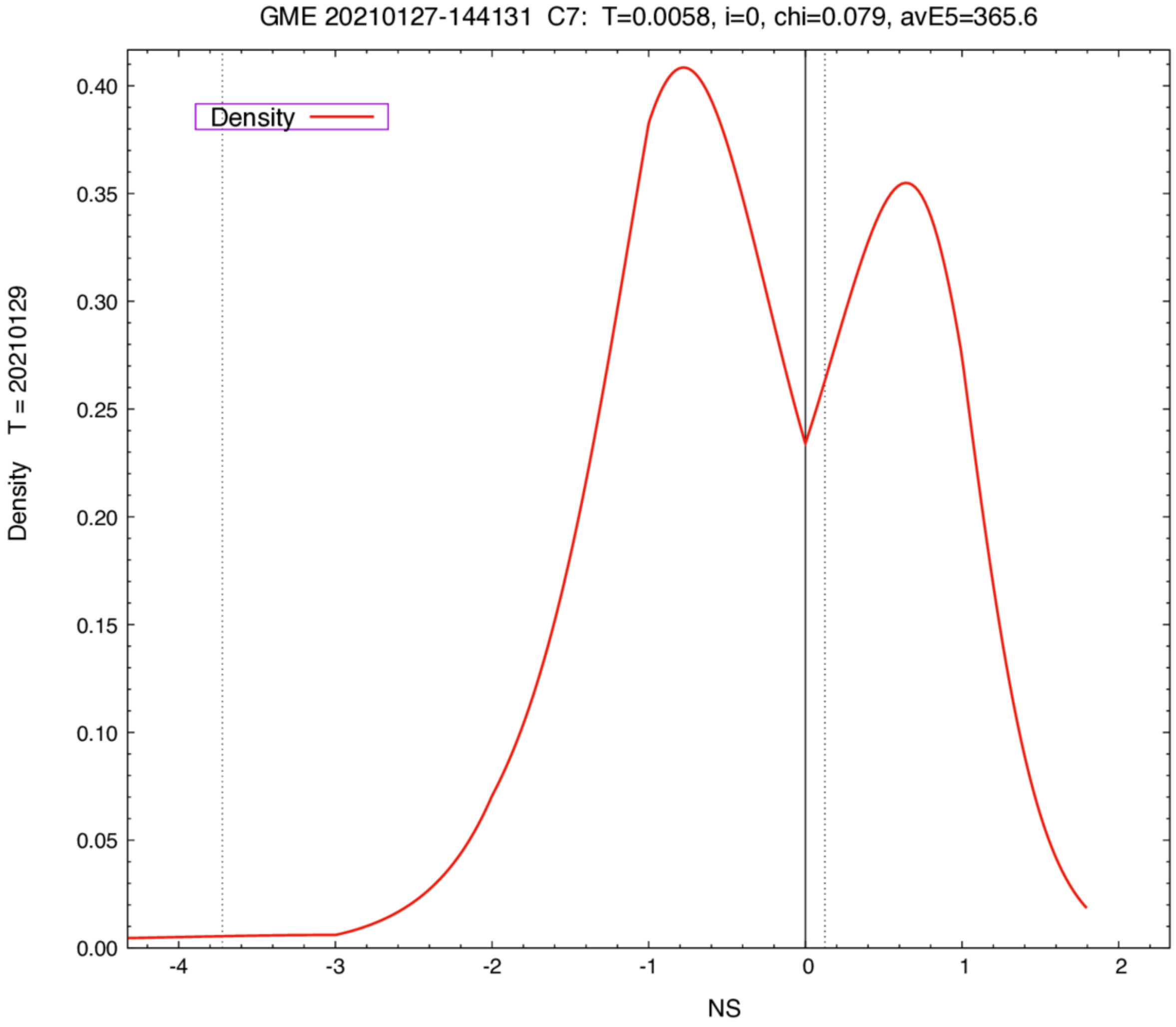
Gamestop 2021-01-27

The day it more than doubled…

Total Variance plot

with error bars

No calendar arb

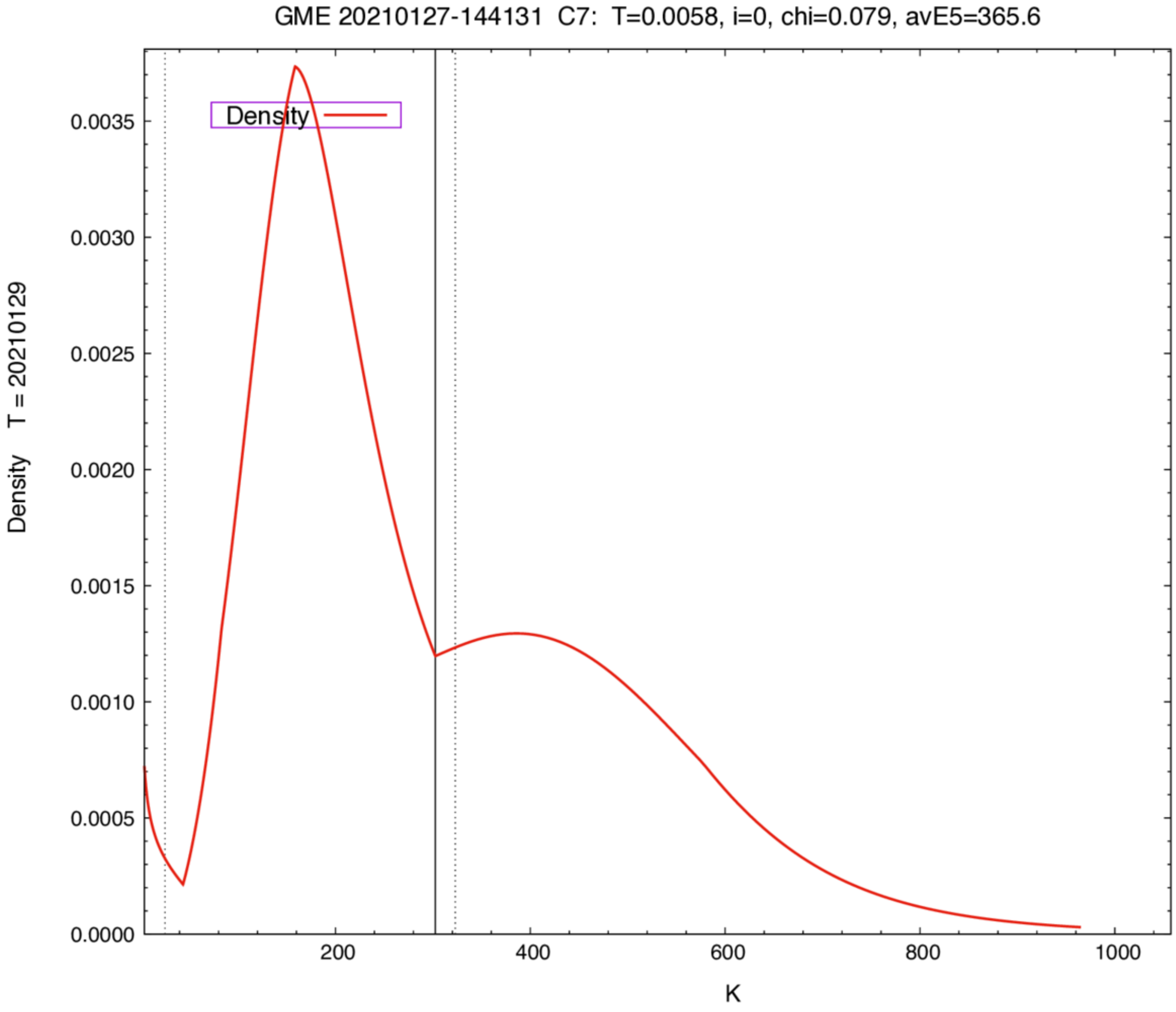


Gamestop 2021-01-27

The day it more than doubled…

Implied density of 1st Term

NS-space



Gamestop 2021-01-27

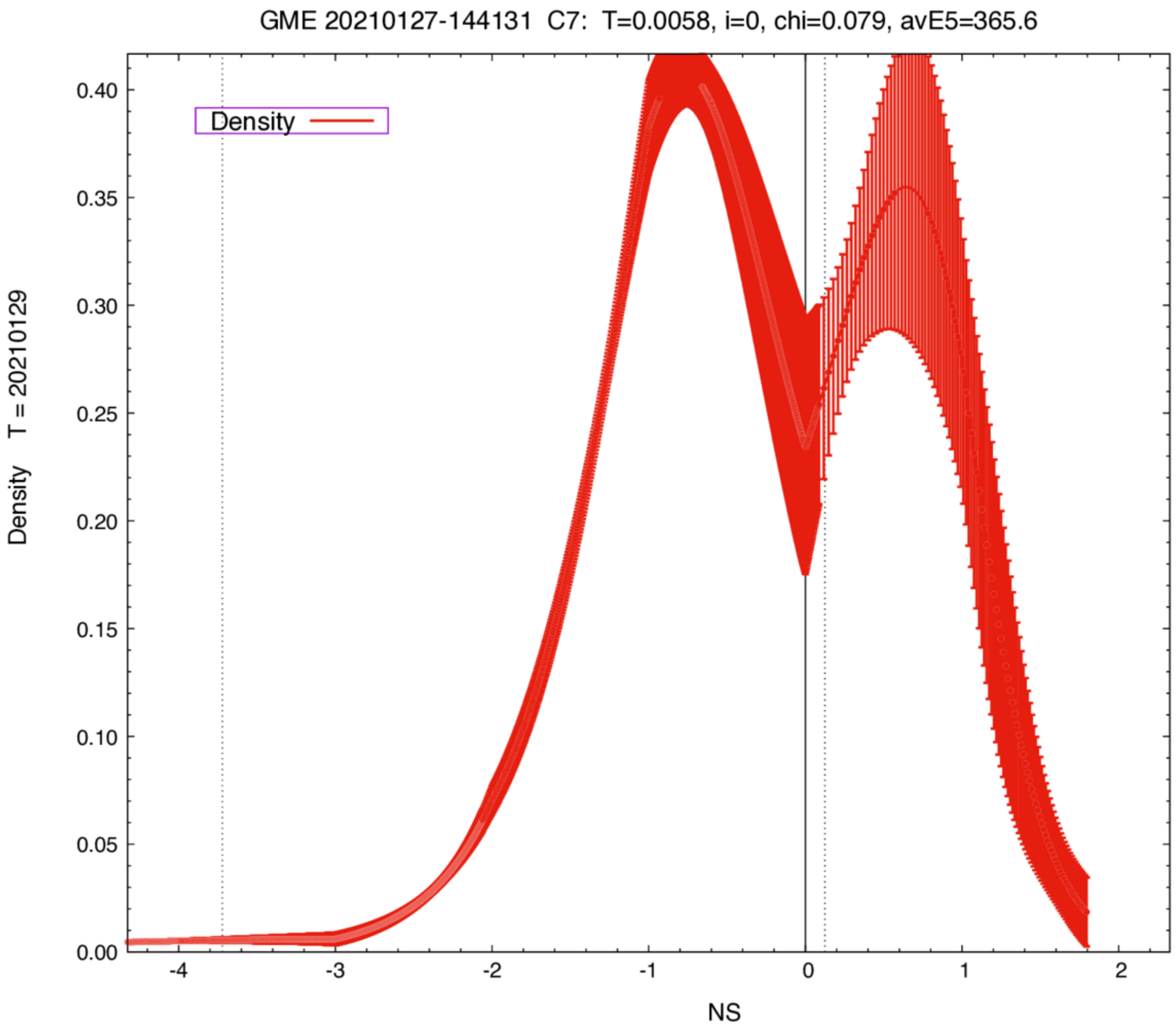
The day it more than doubled…

Implied density of 1st Term

K-space

It could double or half again…

But (almost) no data on call side!?

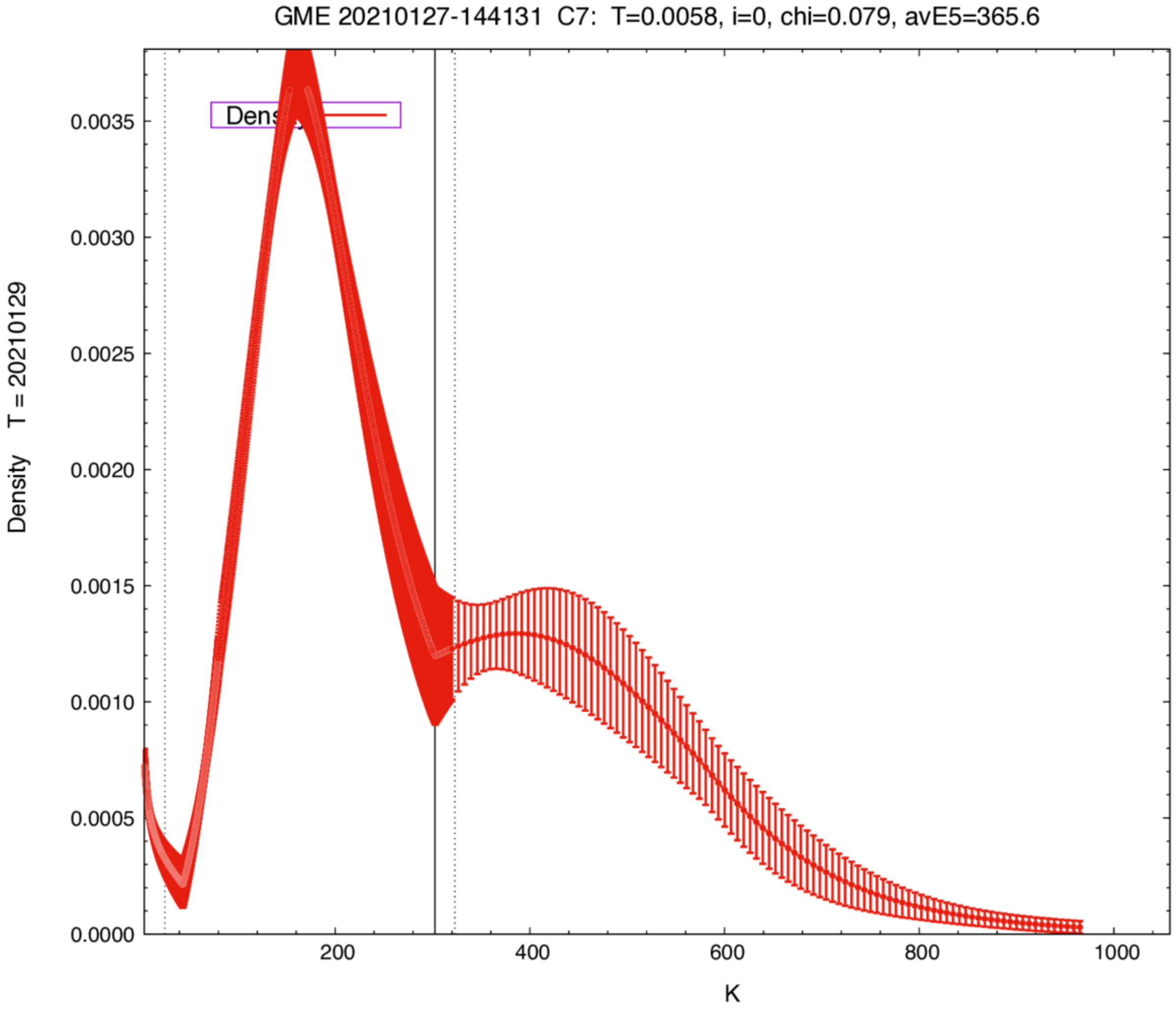


Gamestop 2021-01-27

The day it more than doubled…

Implied density of 1st Term
With error bars!

NS-space



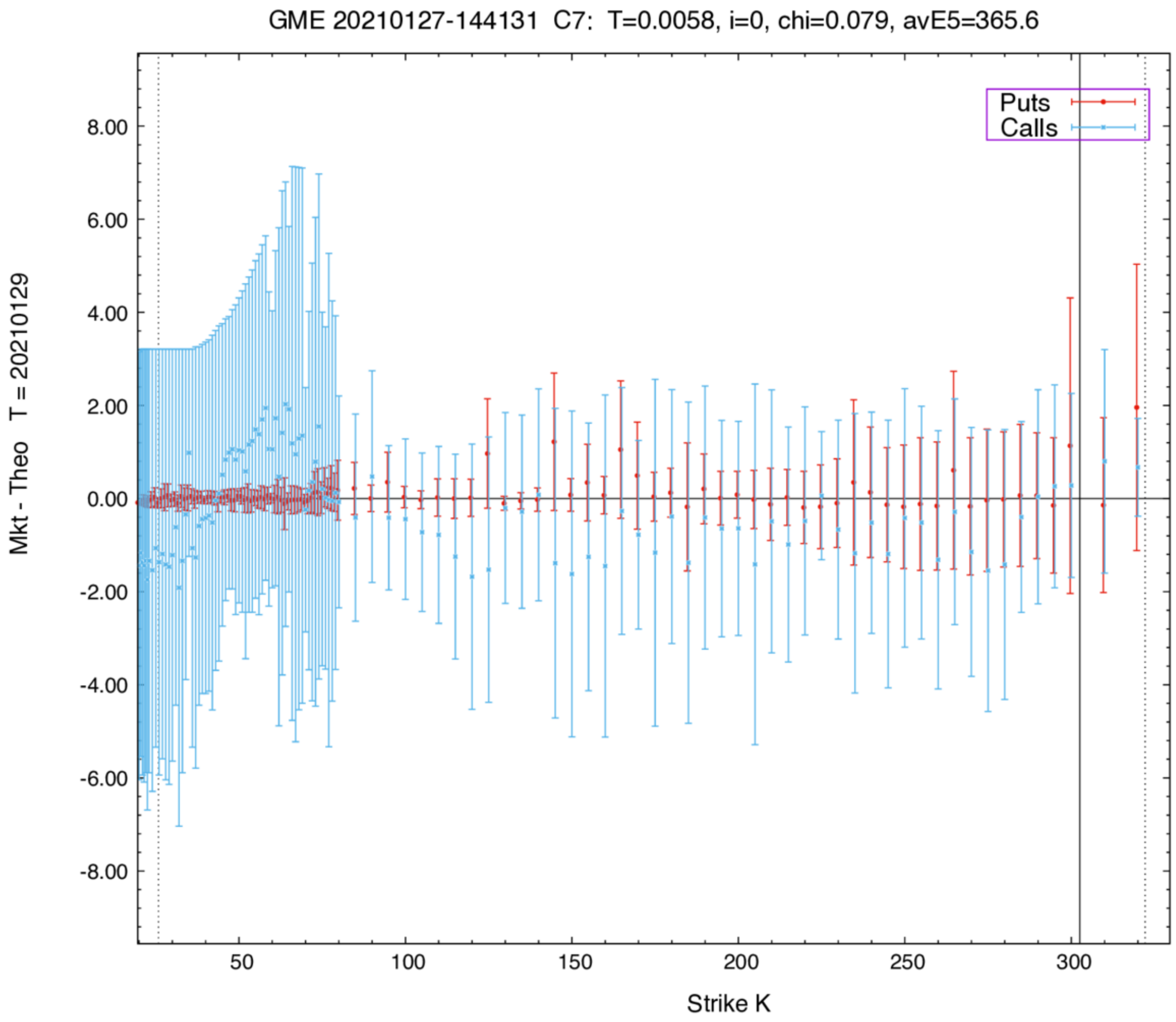
Gamestop 2021-01-27

The day it more than doubled…

Implied density of 1st Term
With error bars!

K-space

How large are systematic error bars on call side??

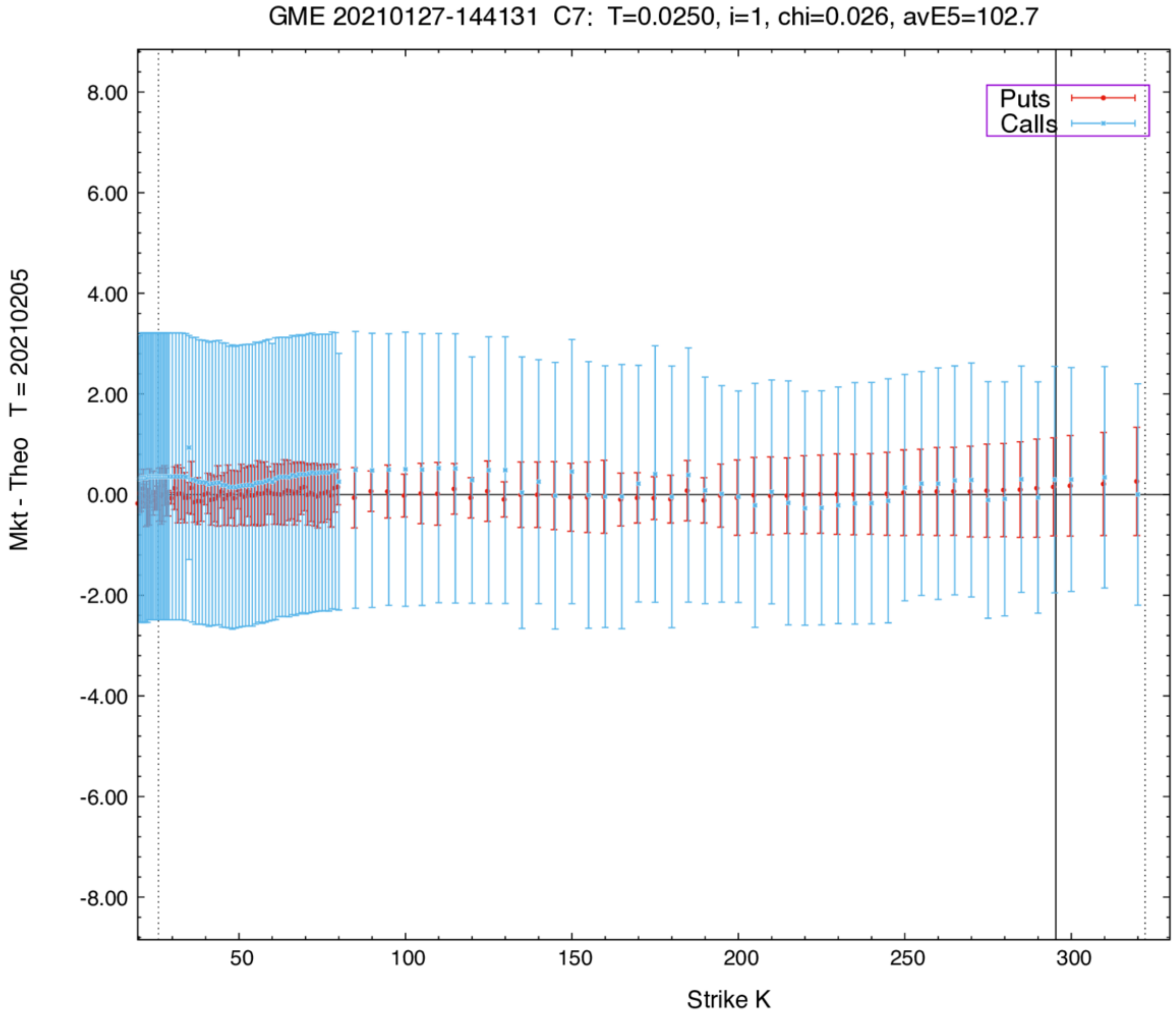


Gamestop 2021-01-27

The day it more than doubled…

Implied vols of 1st Term: P & C

PriceDiff plot:
The ultimate truth

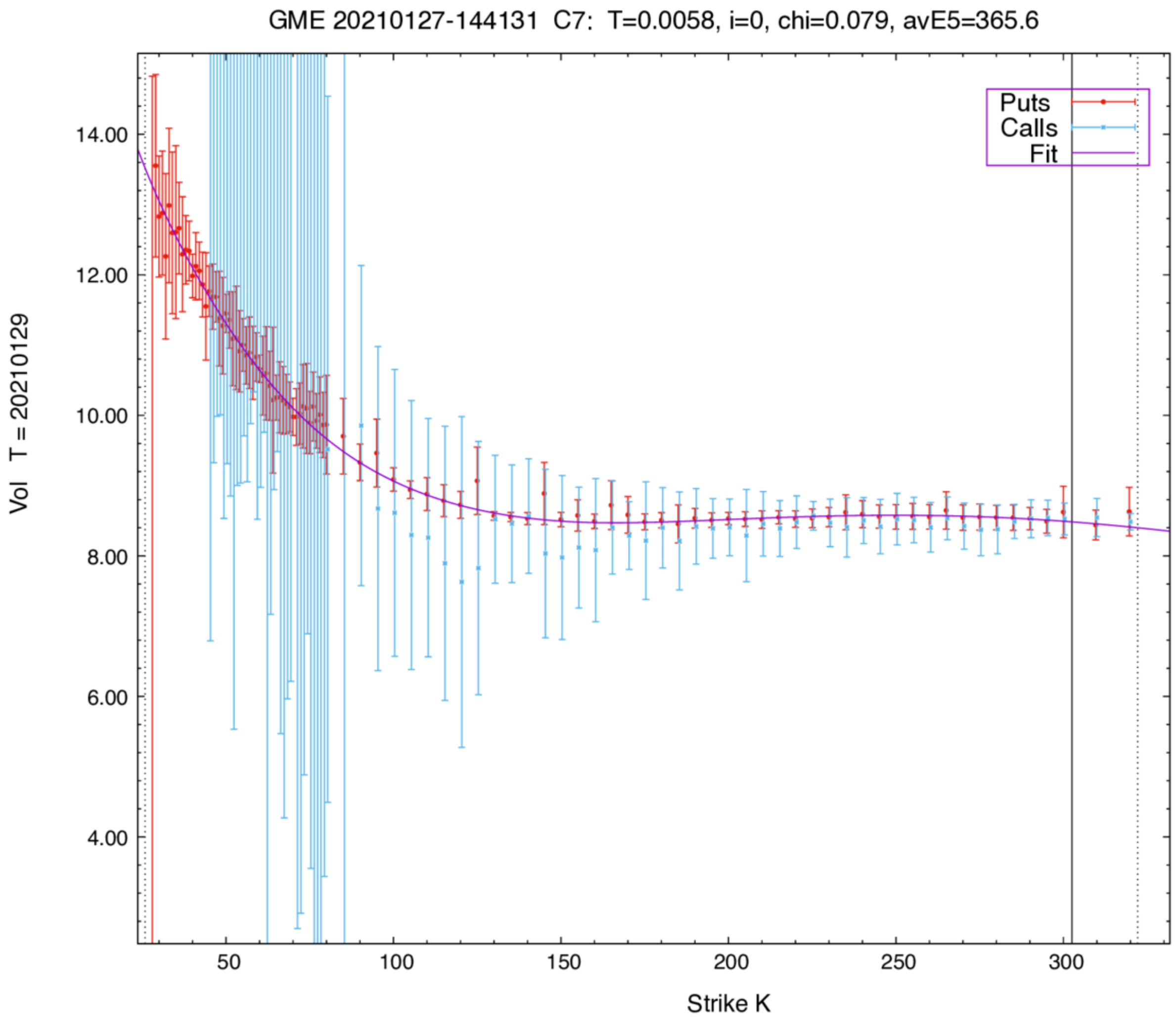


Gamestop 2021-01-27

The day it more than doubled…

Implied vols of 2nd Term: P & C

PriceDiff plot:
The ultimate truth

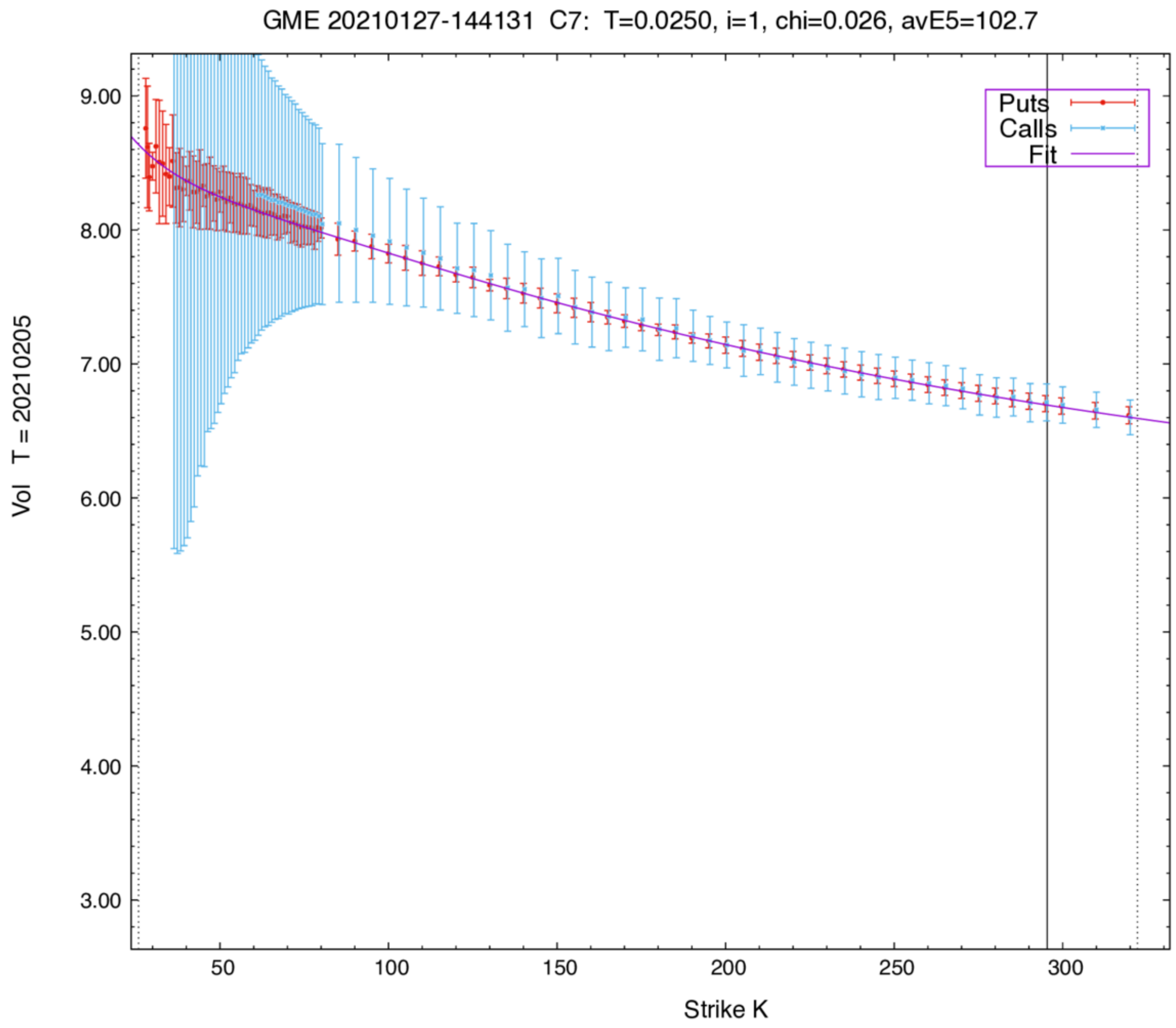


Gamestop 2021-01-27

The day it more than doubled…

Implied vols of 1st Term: P & C

American PCP holds

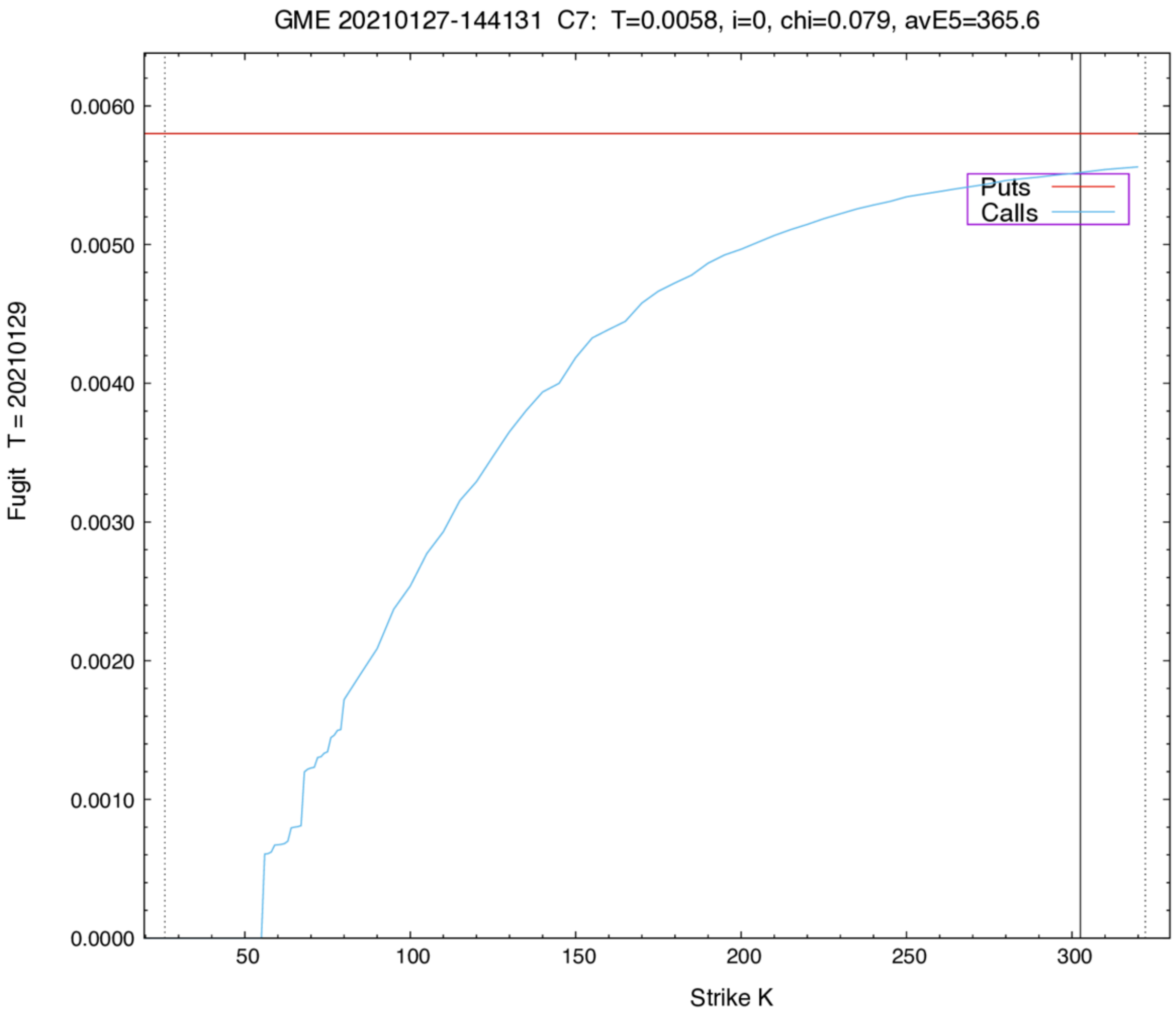


Gamestop 2021-01-27

The day it more than doubled…

Implied vols of 2nd Term: P & C

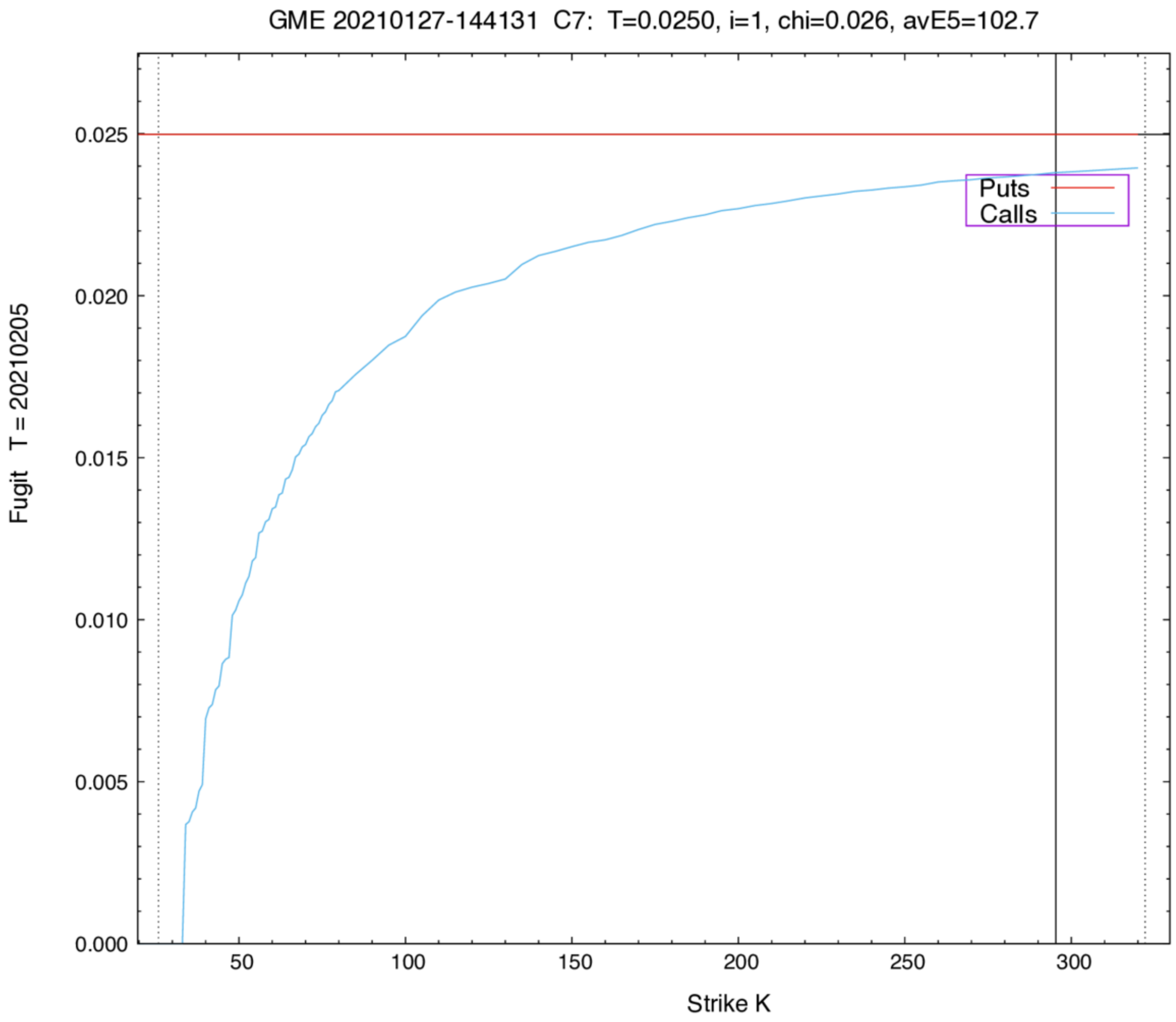
American PCP holds



Gamestop 2021-01-27

The day it more than doubled…

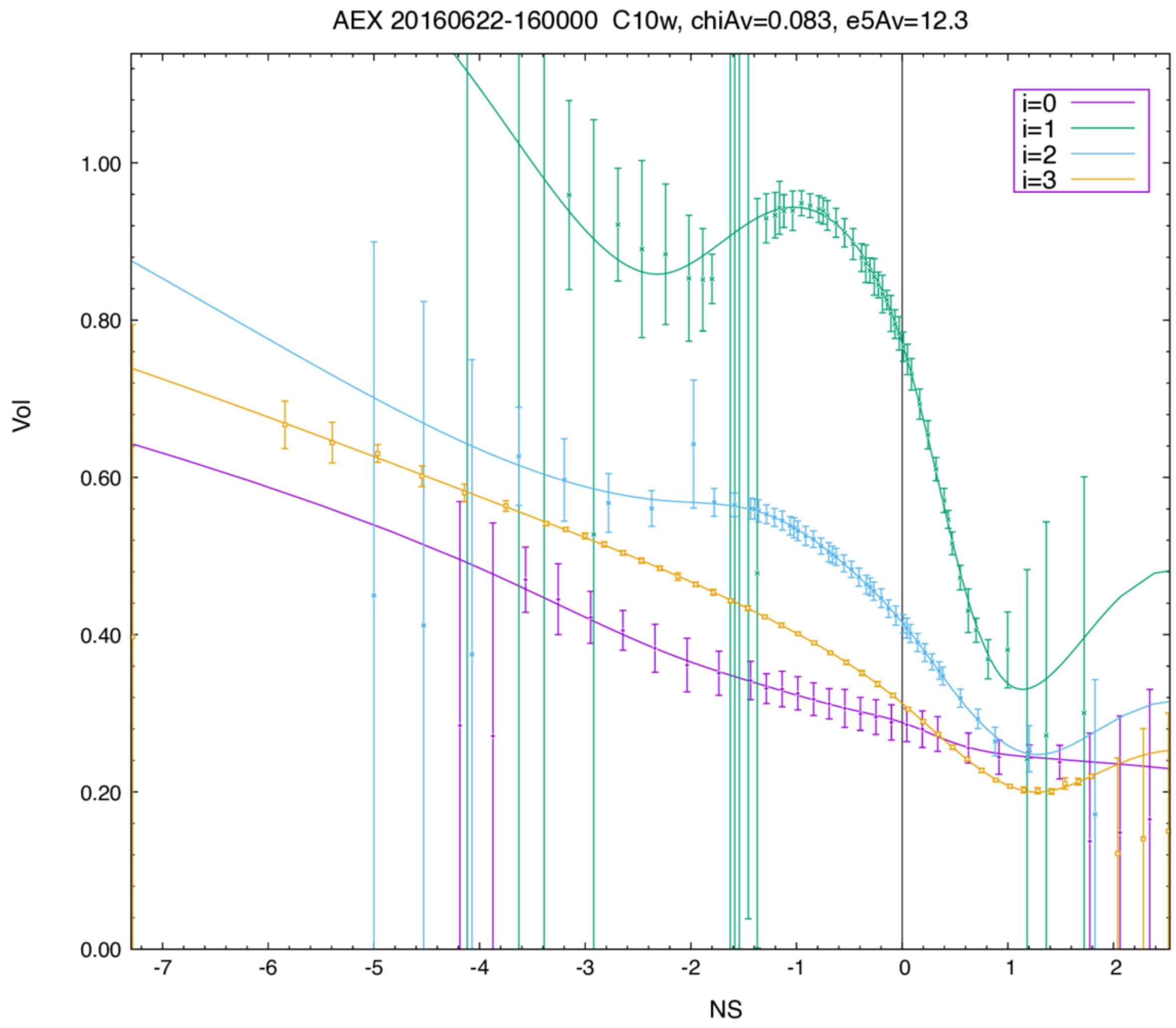
Fugit of 1st Term



Gamestop 2021-01-27

The day it more than doubled…

Fugit of 2nd Term

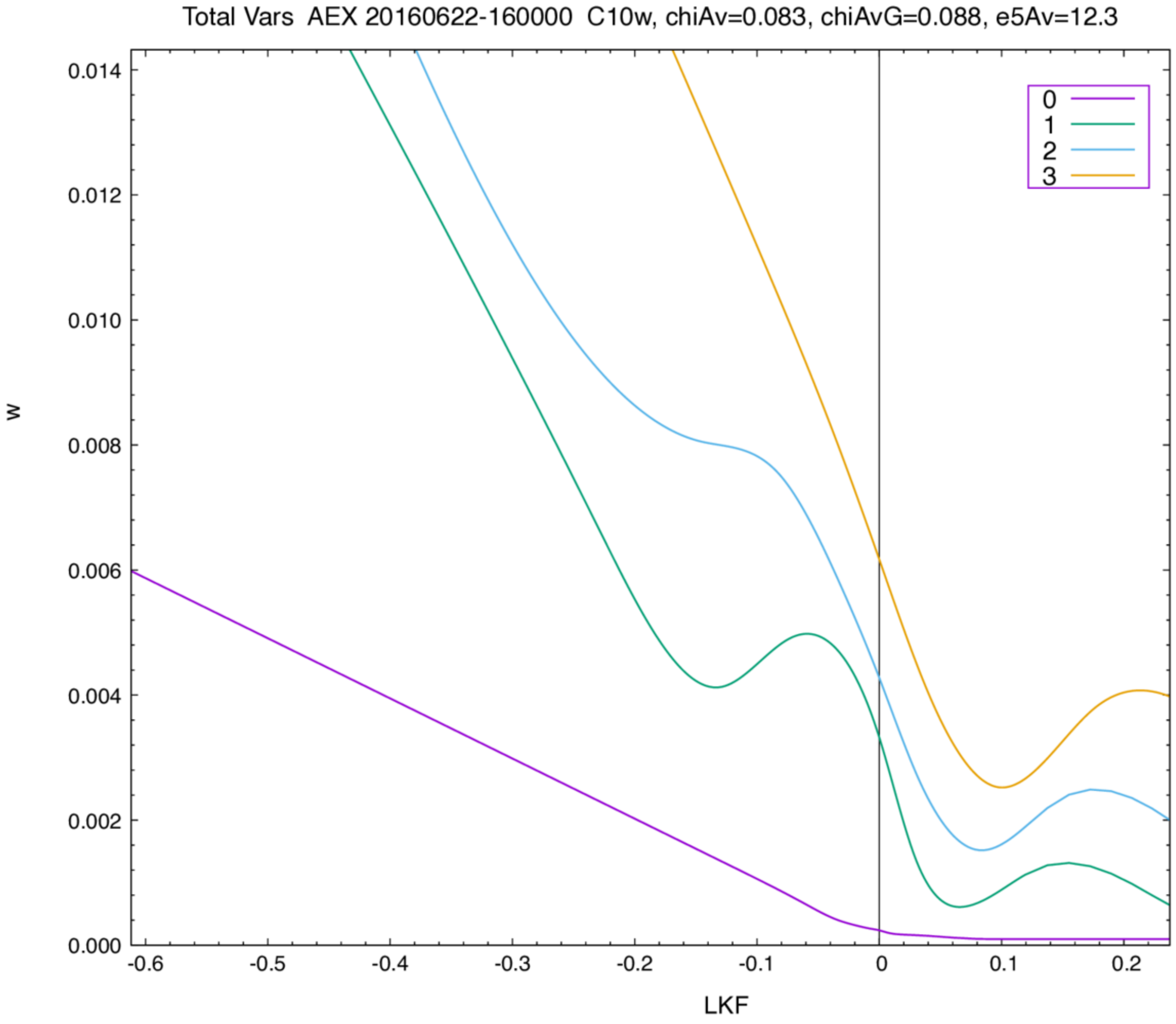


AEX 2016-06-22

Day before Brexit!

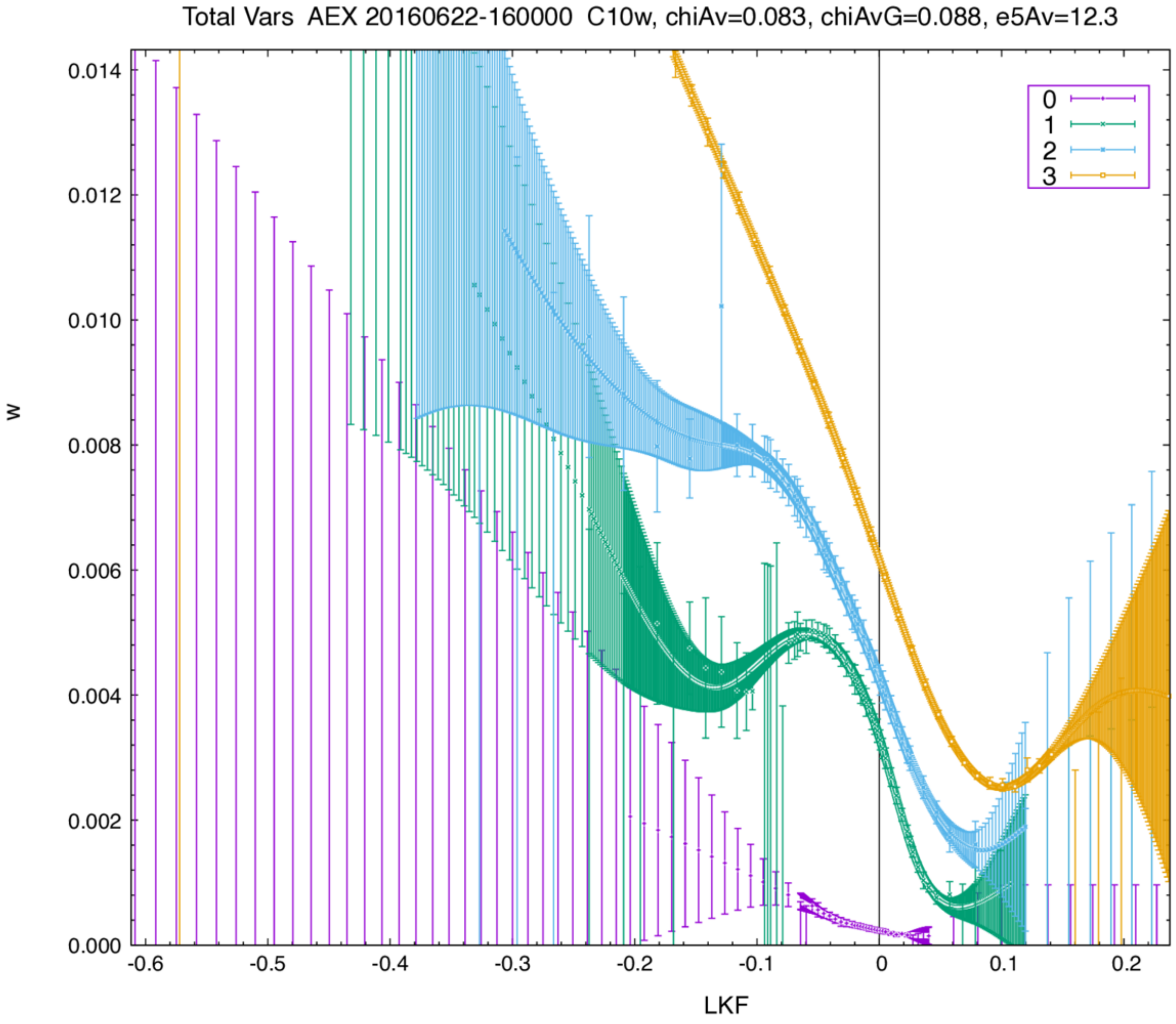
Vol vs NS

$$z := \mathrm{NS} := \frac{\ln(K/F)}{\sigma_0\sqrt{T}}$$



Fitting **AEX** on day before Brexit

Total Var plot

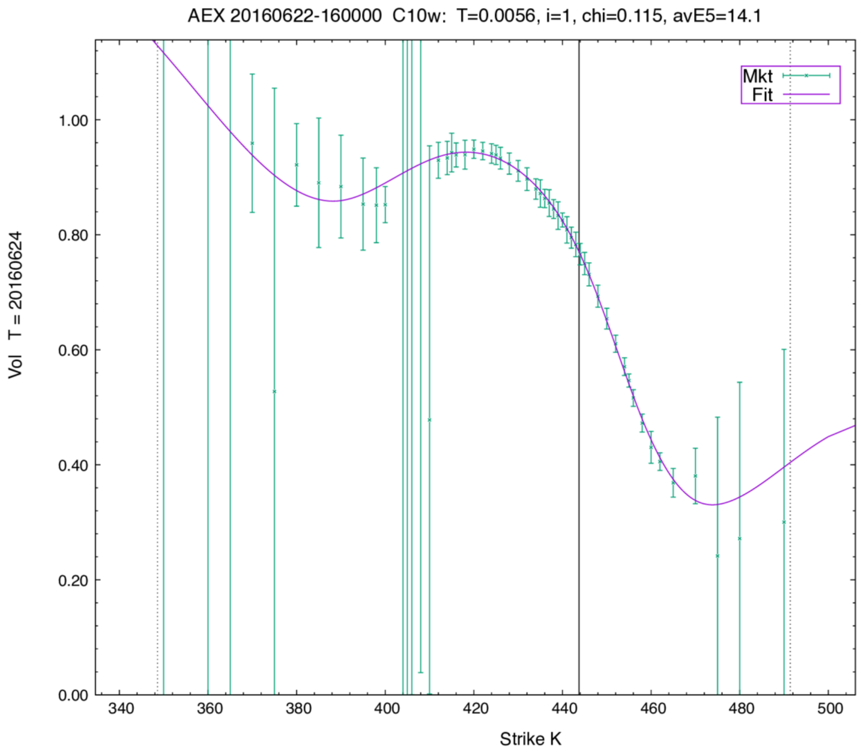


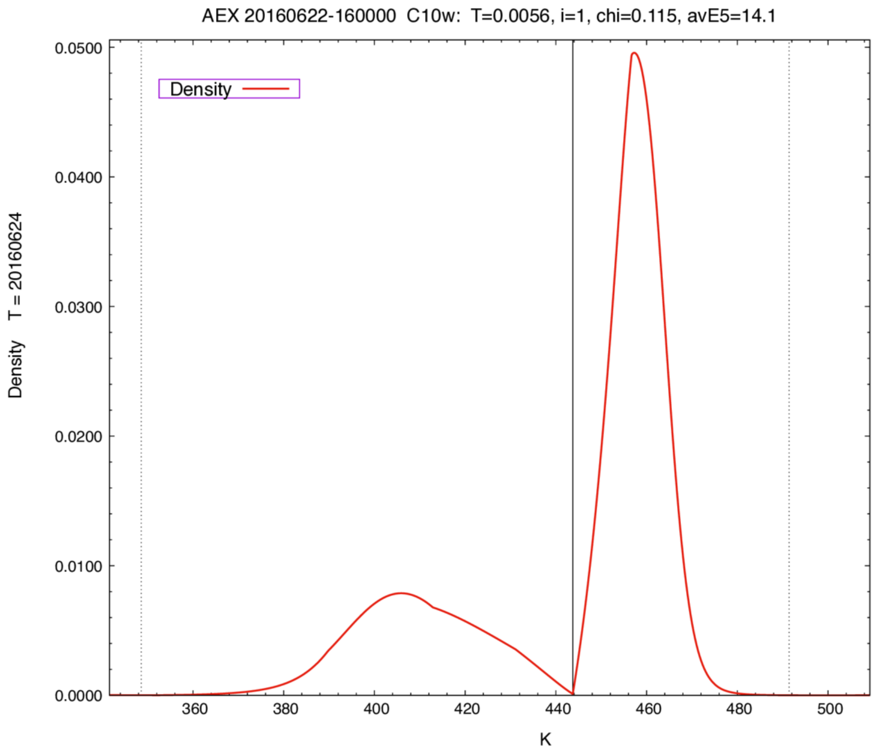
Fitting **AEX** on day before Brexit

Total Var plot
with error bars

AEX on day before Brexit vote:

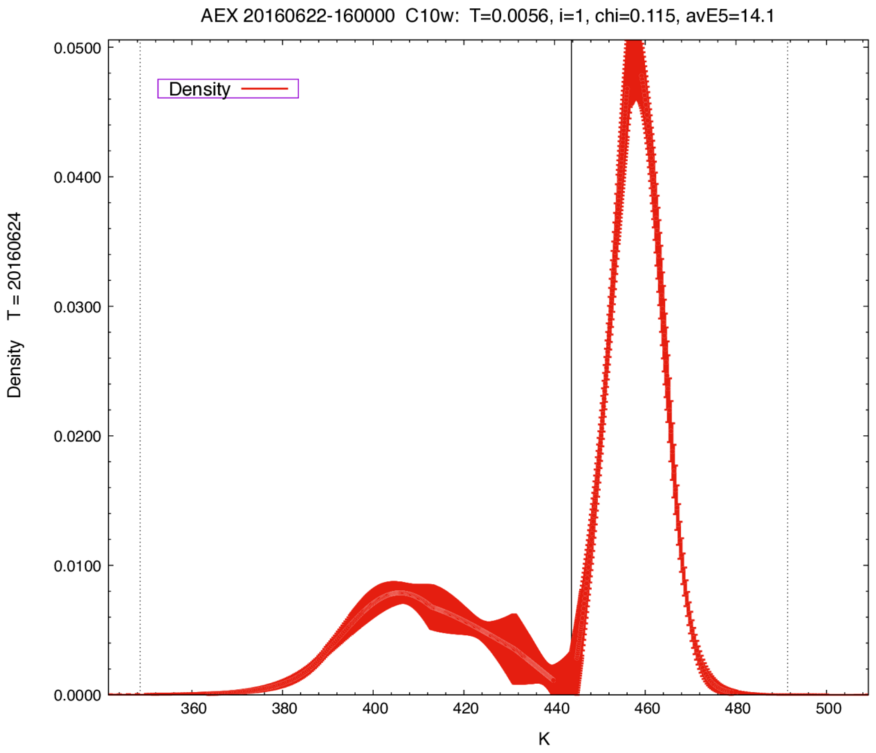
T=2d, vols and implied density

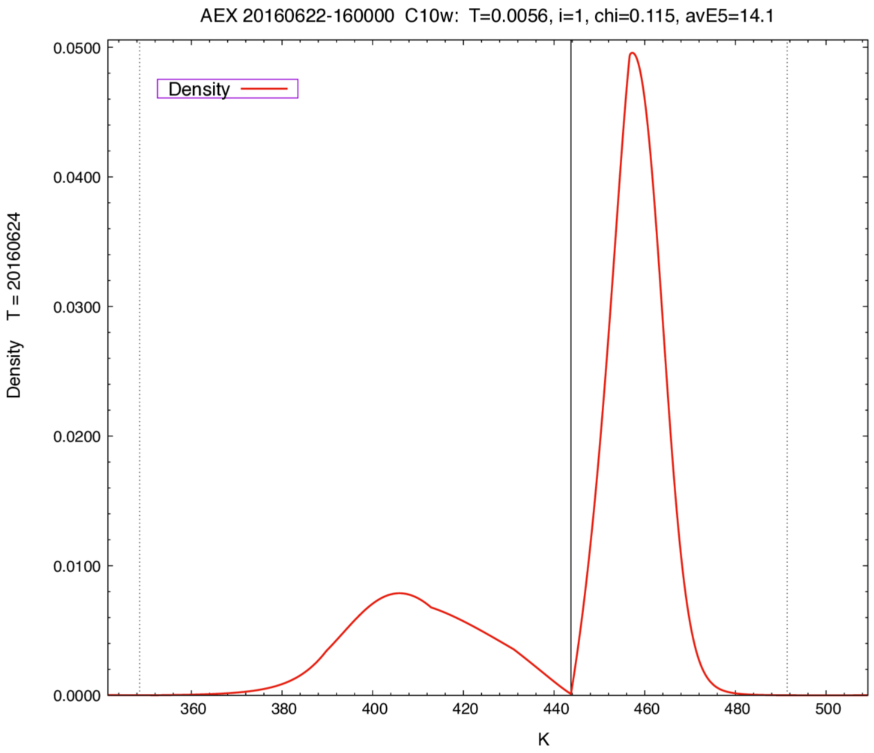


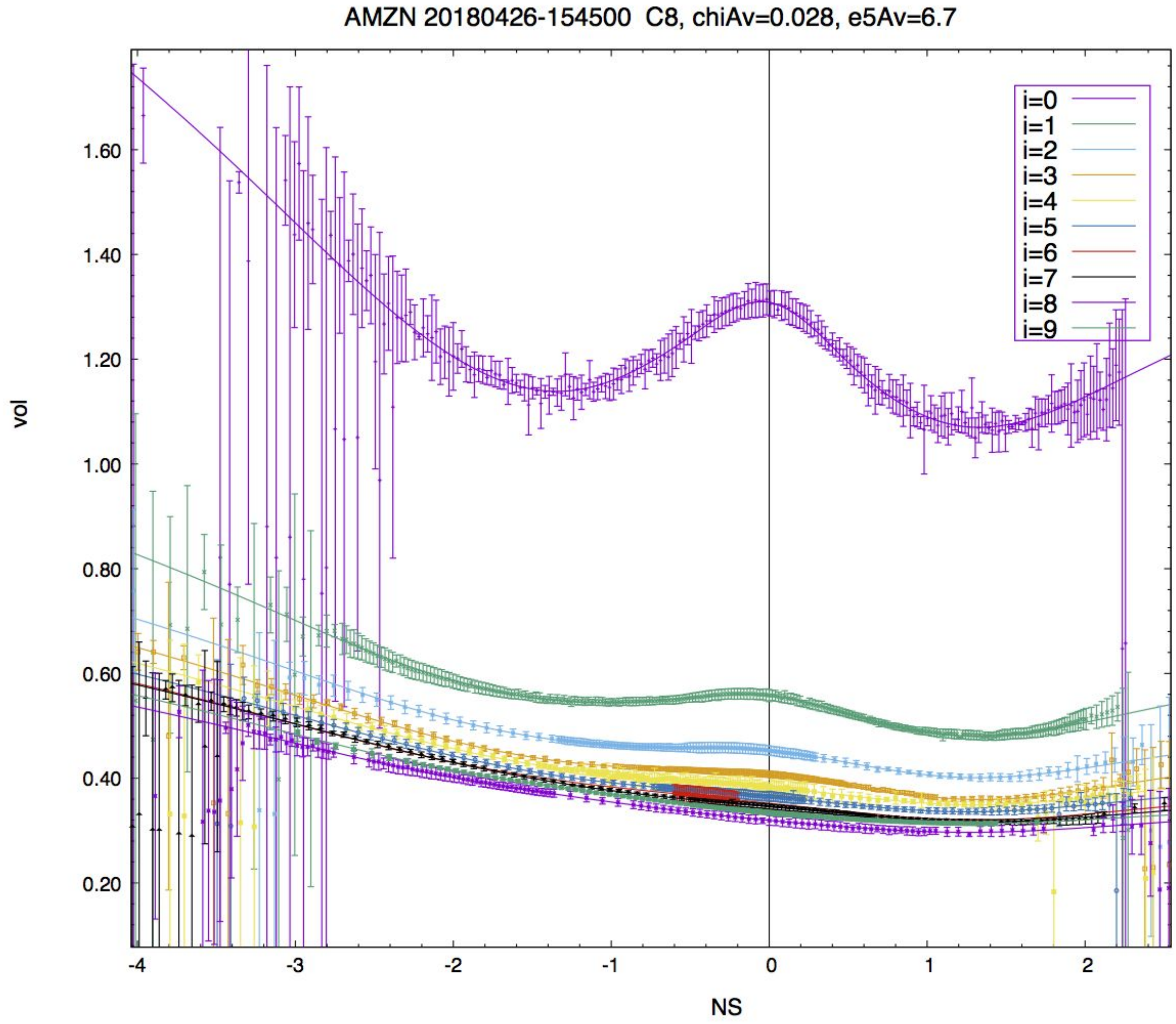


AEX on day before Brexit vote:

T=2d, implied density with error bars (AEX dropped 5.9%)







AMZN 2018-04-26 earnings day

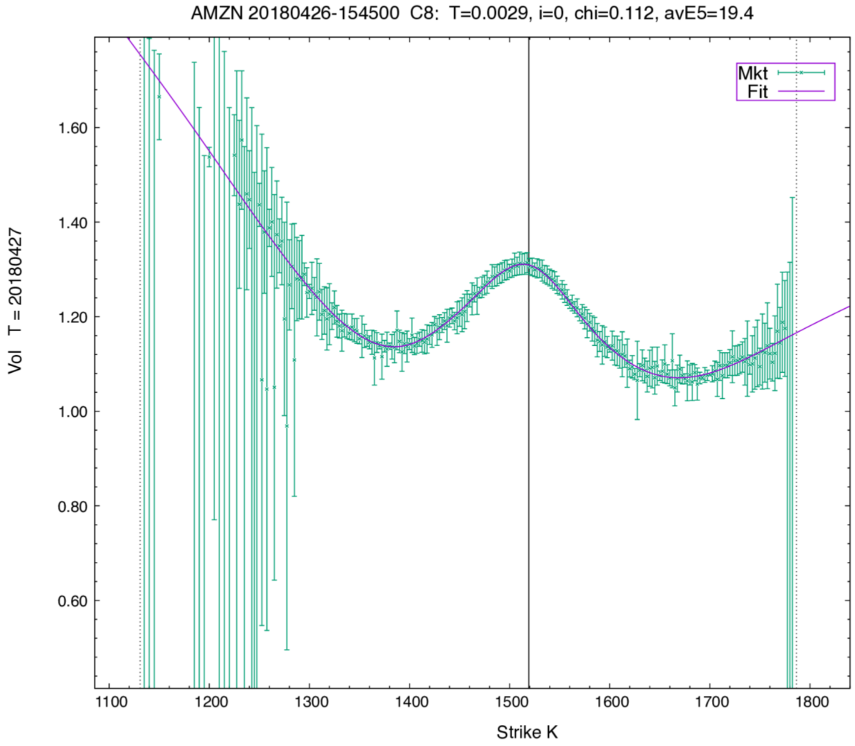
C8 Vol vs NS

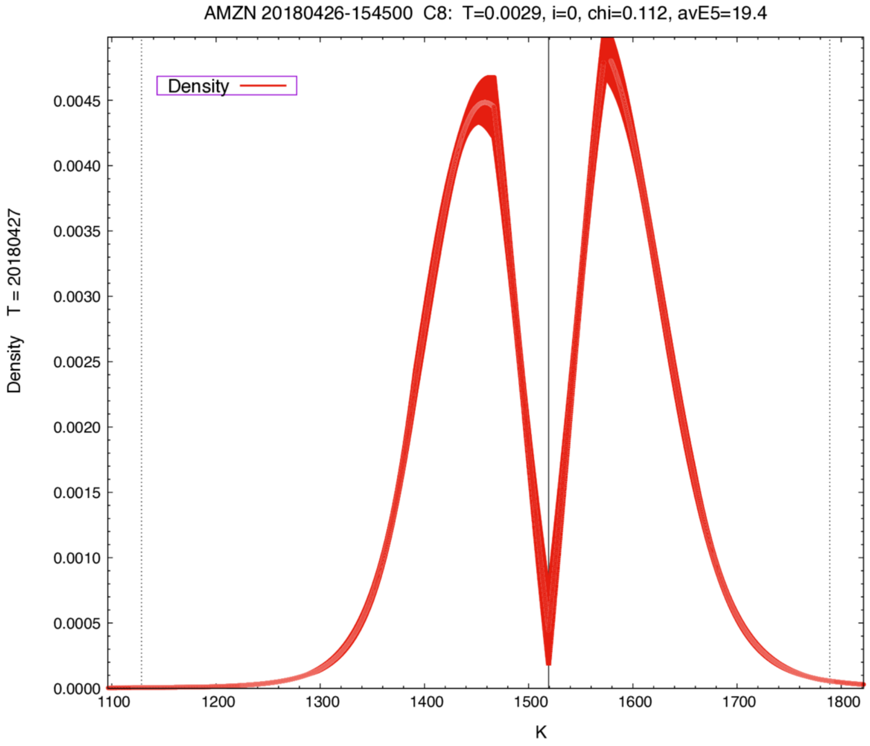
Interesting Thursday: Earnings, new weekly listed, etc.

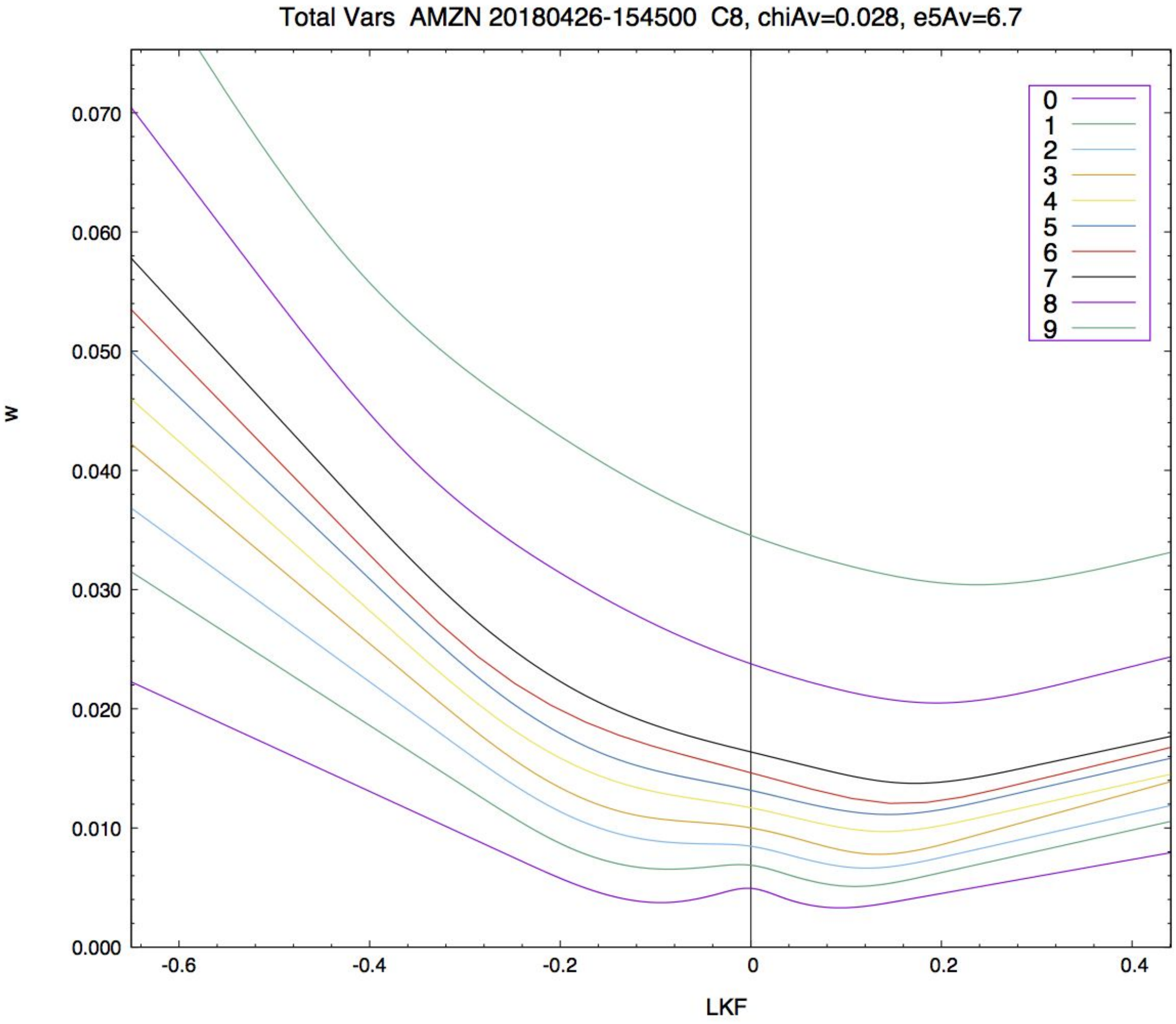
$$z := \mathrm{NS} := \frac{\ln(K/F)}{\sigma_0\sqrt{T}}$$

AMZN 2018-04-26 earnings day:

T=1d, vols and implied density — most bimodal density ever!







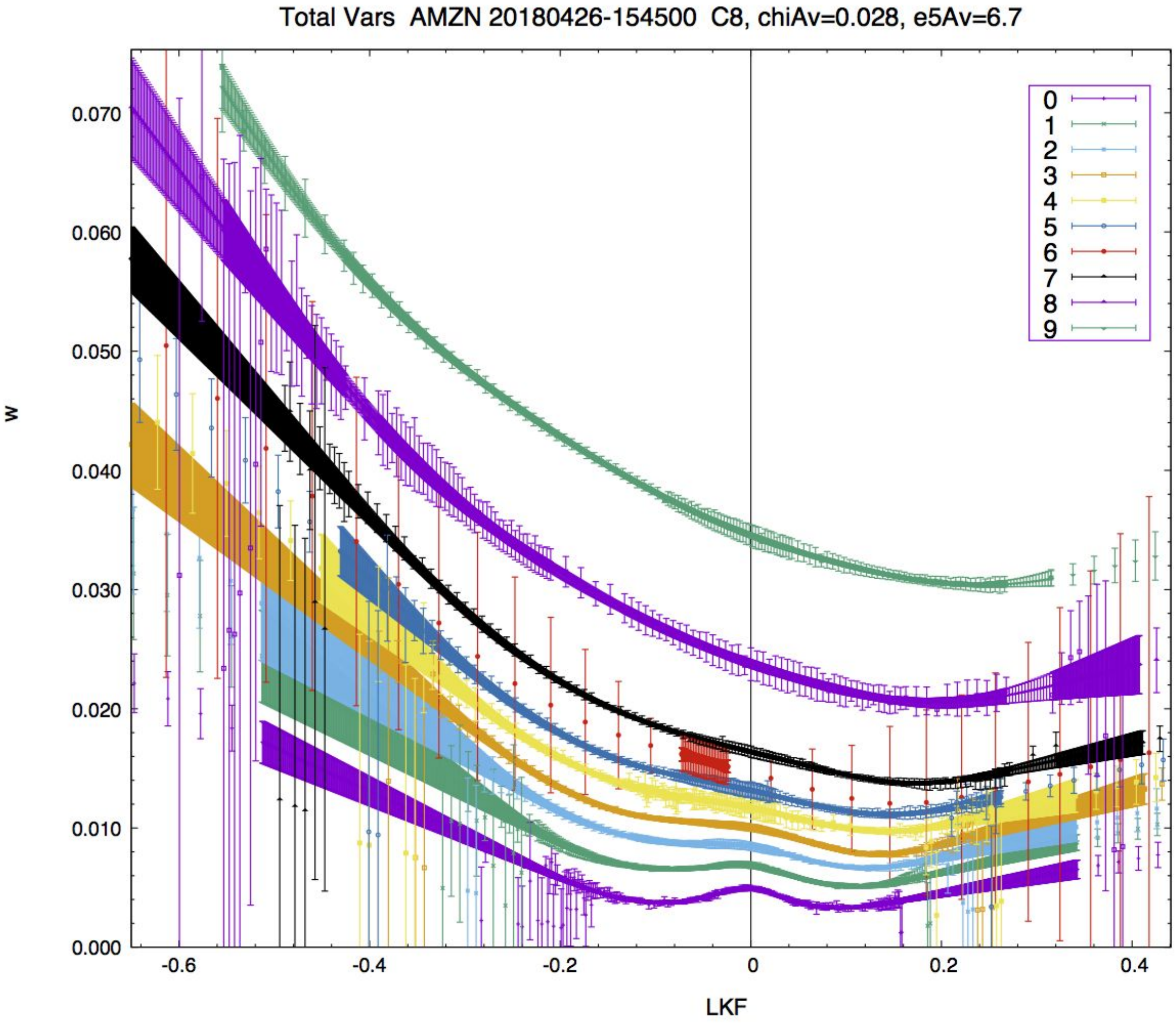
AMZN 2018-04-26 earnings day

C8 total variance plot

First 10 terms

No calendar arbitrage! (Or butterfly…)

Interesting Thursday: Earnings, new weekly listed (**i=6**), etc.

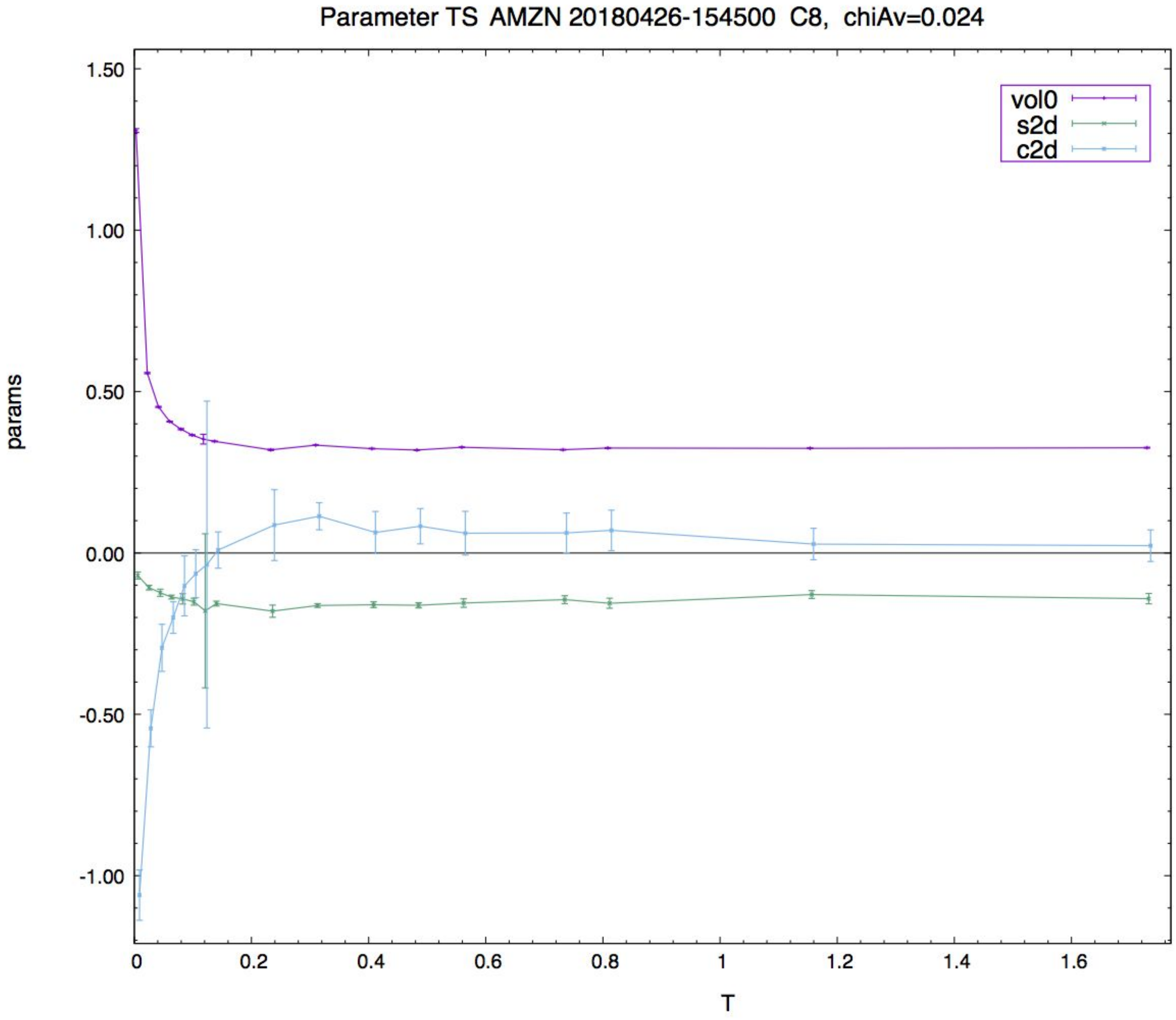


AMZN 2018-04-26 earnings day

C8 total variance plot

First 10 terms, with errors bars

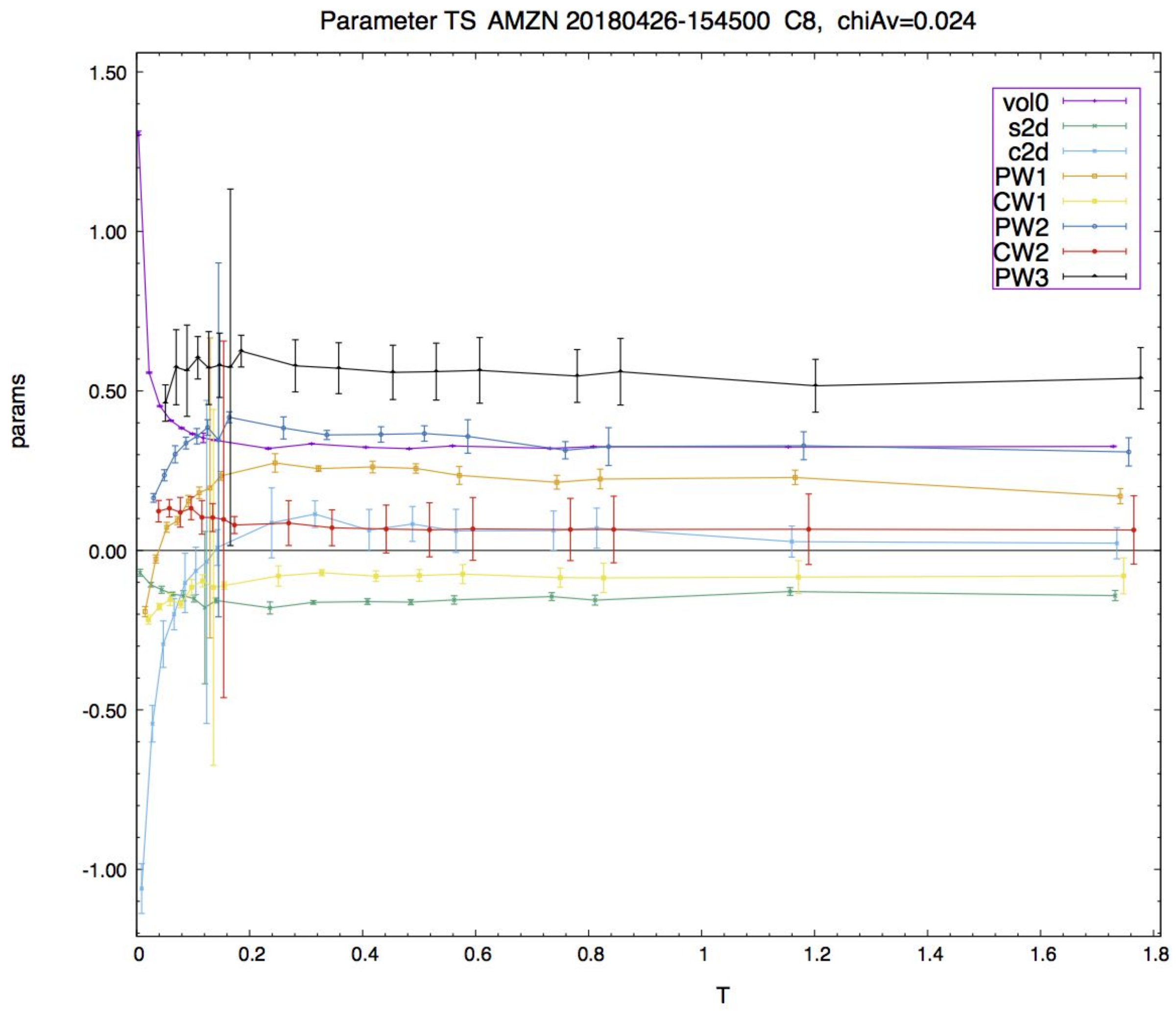
Interesting Thursday: Earnings, new weekly listed ($i=6$), etc.



AMZN 2018-04-26
earnings day

C8 parameter term-structure
First 3: vol0, s2, c2

Essentially flat shape params after 3m



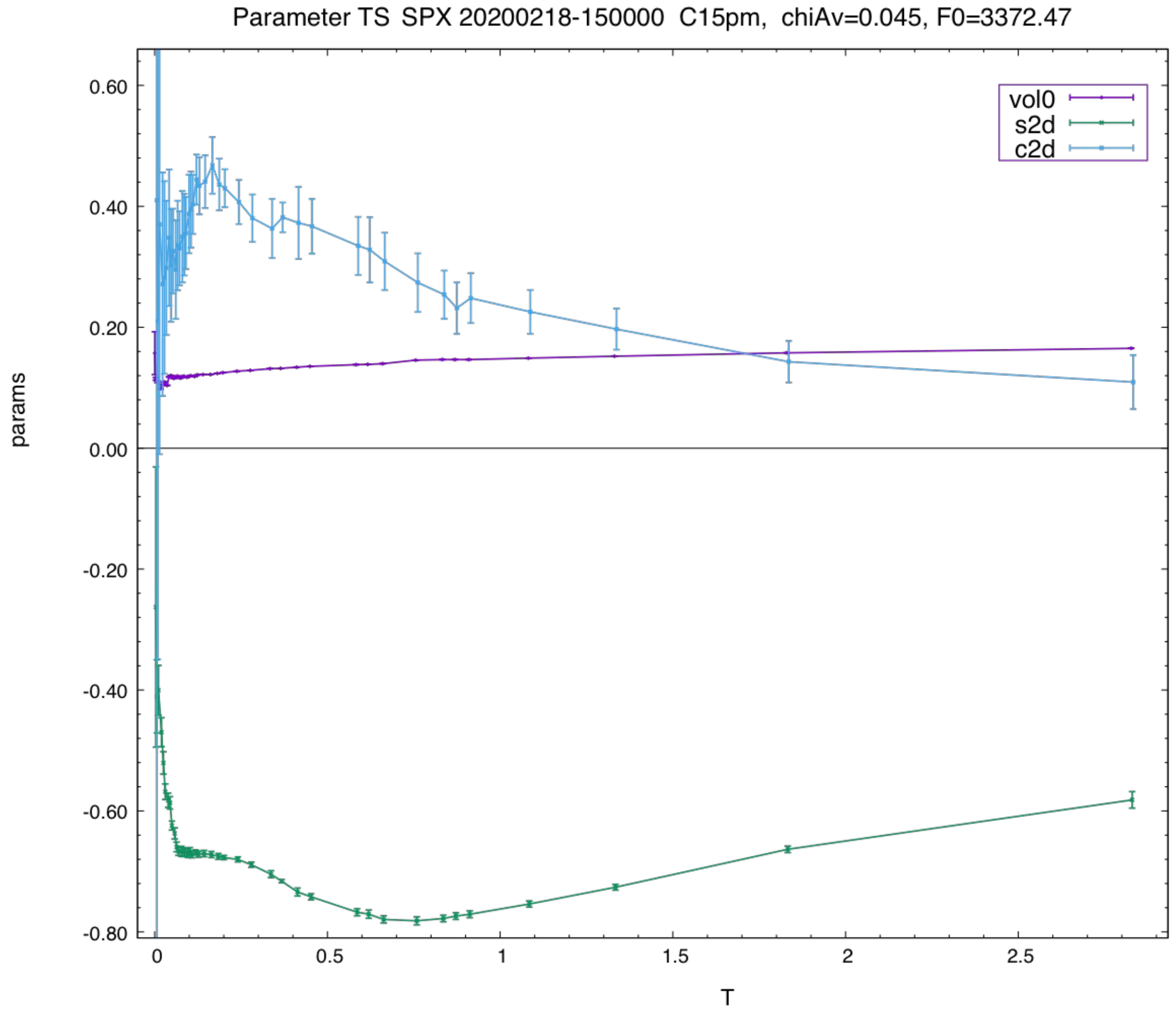
AMZN 2018-04-26 earnings day

C8 parameter term-structure

Essentially flat shape params after 3m

SPX during the covid crash of March 2020

- SPX vol curves and surfaces had unprecedented shapes.
- Supposedly even some Tier 1 bank(s) didn’t manage to produce a tradable SPX surface for 2 days (e.g. arbitrage-free).
- But the options market functioned perfectly fine at all times and those “funky” shapes reflected quite precise & consistent forward-looking views.



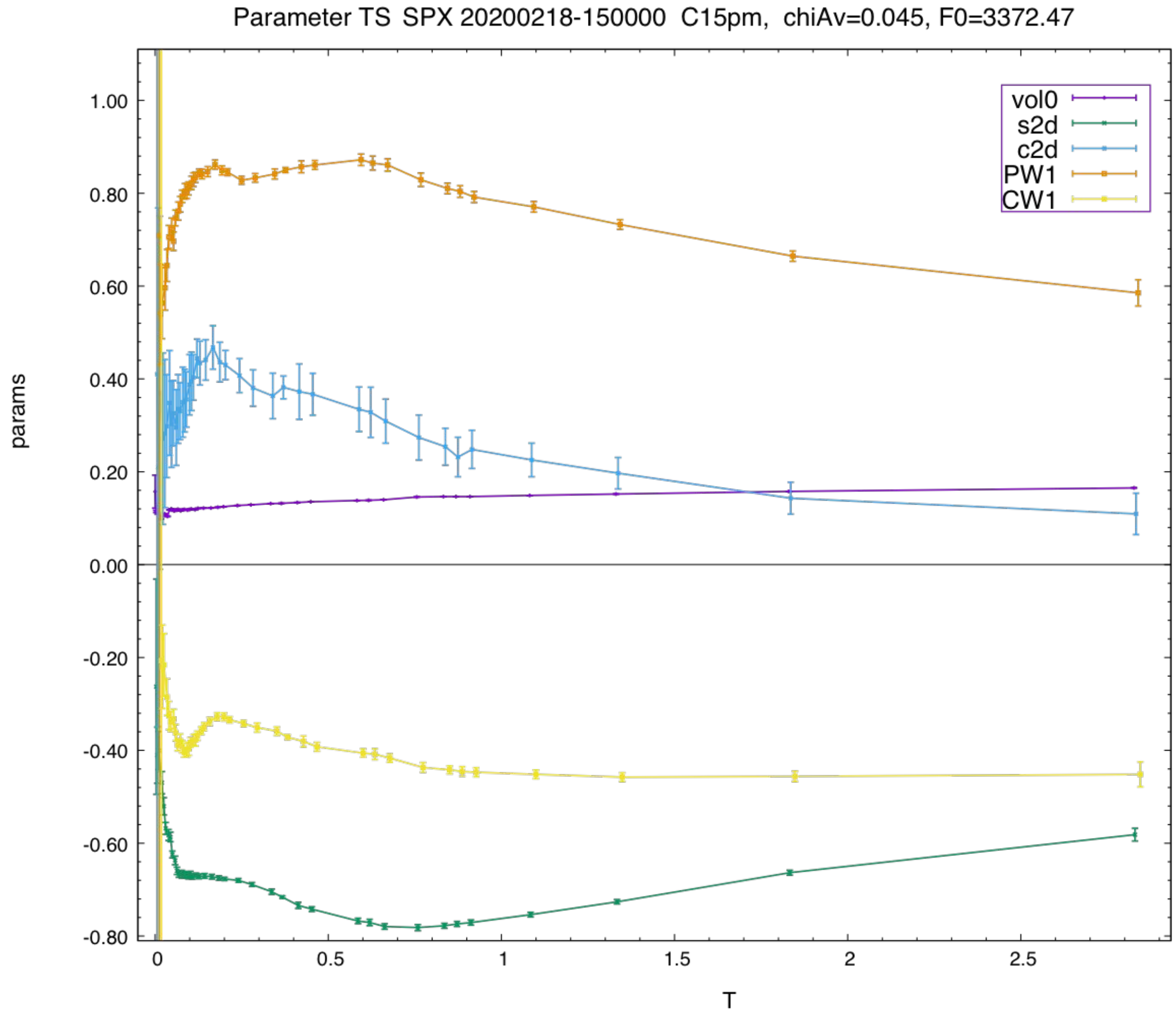
For reference: 3w before big covid crash…

SPX 20200218 15:00

C15PM Param Term-Structure

First 3 params…

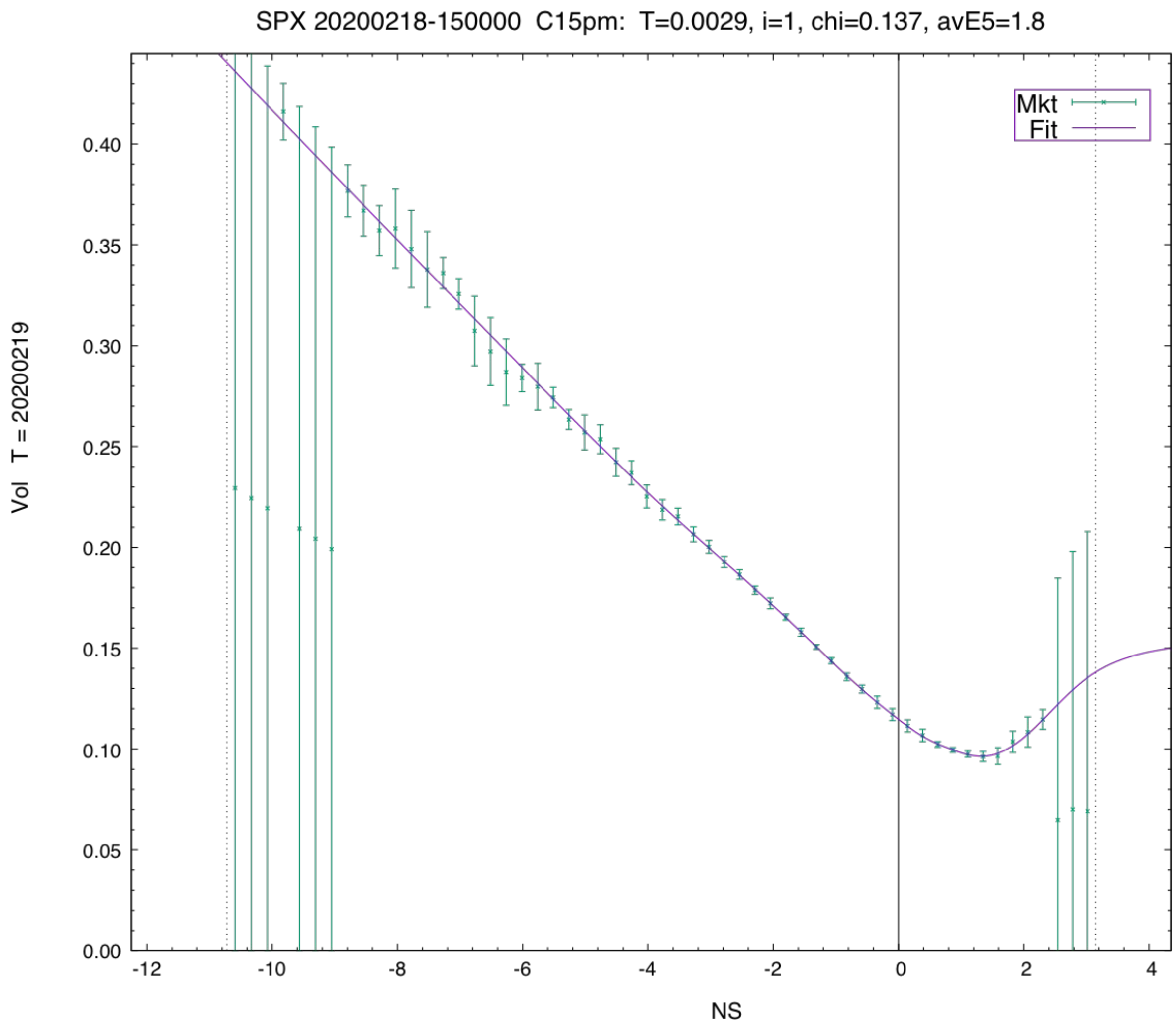
s2(T) a bit unusual…



SPX 20200218 15:00

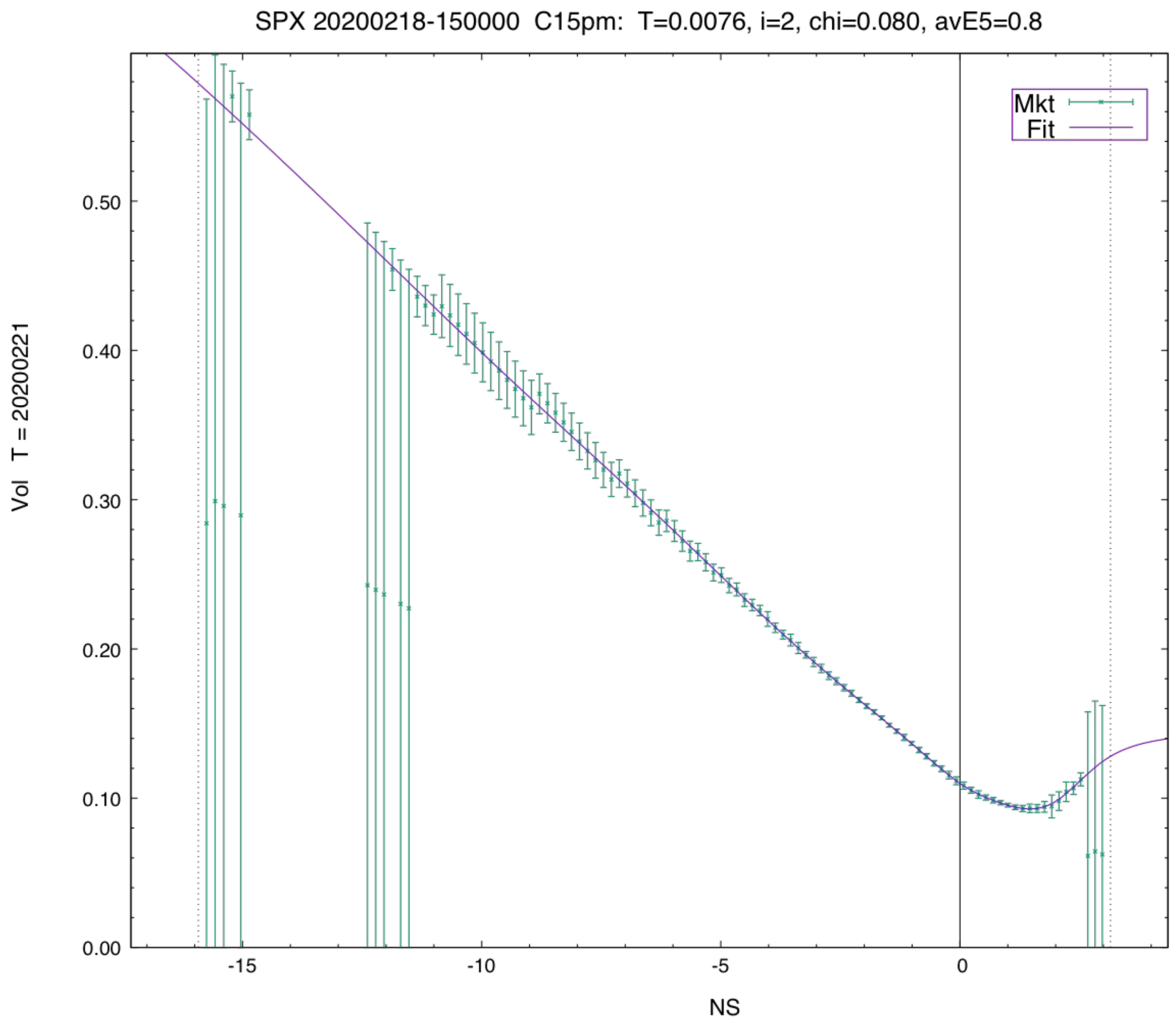
C15PM Param Term-Structure

First 5 params… meaning?



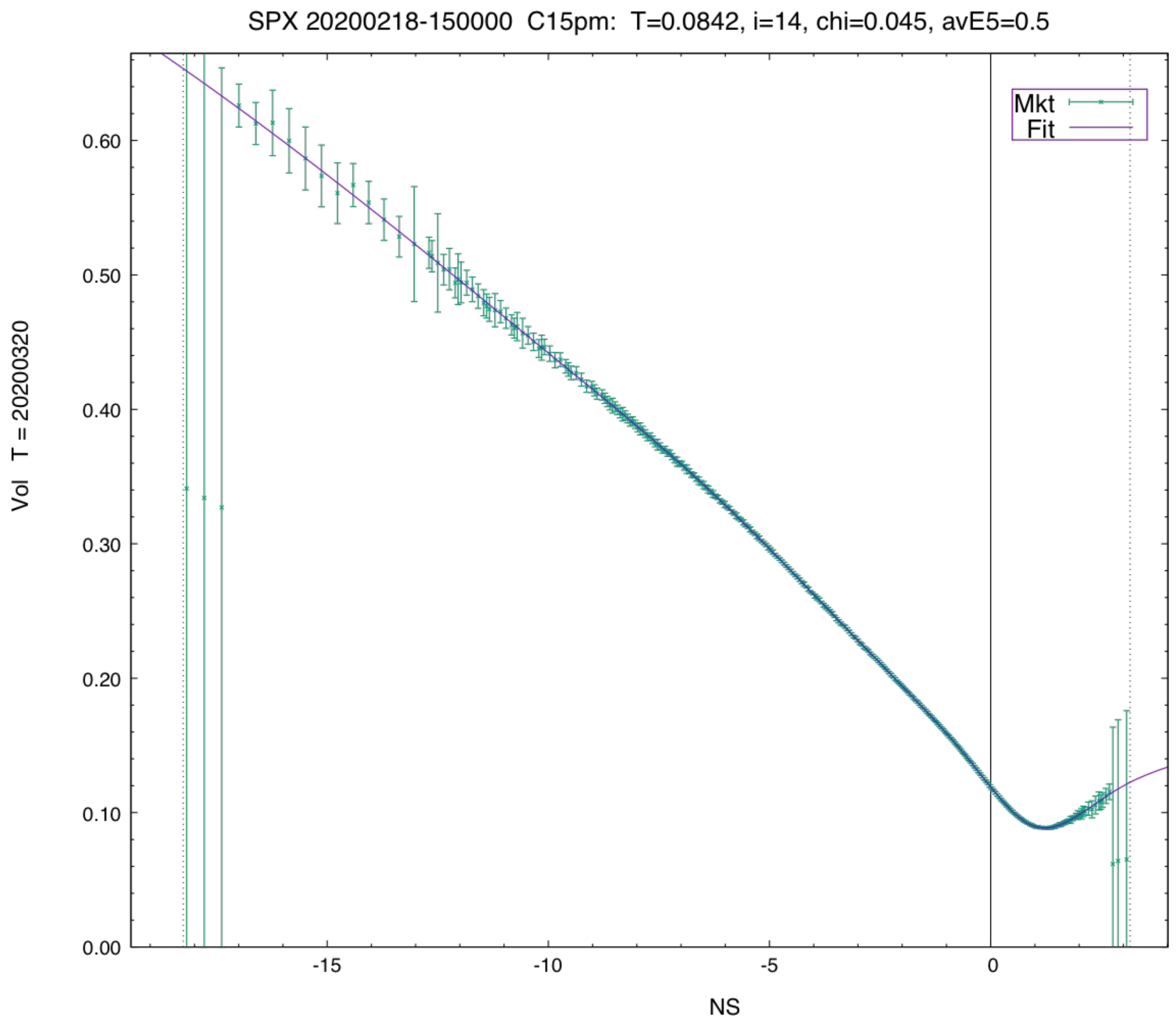
SPX 20200218 15:00

C15PM T = 1d



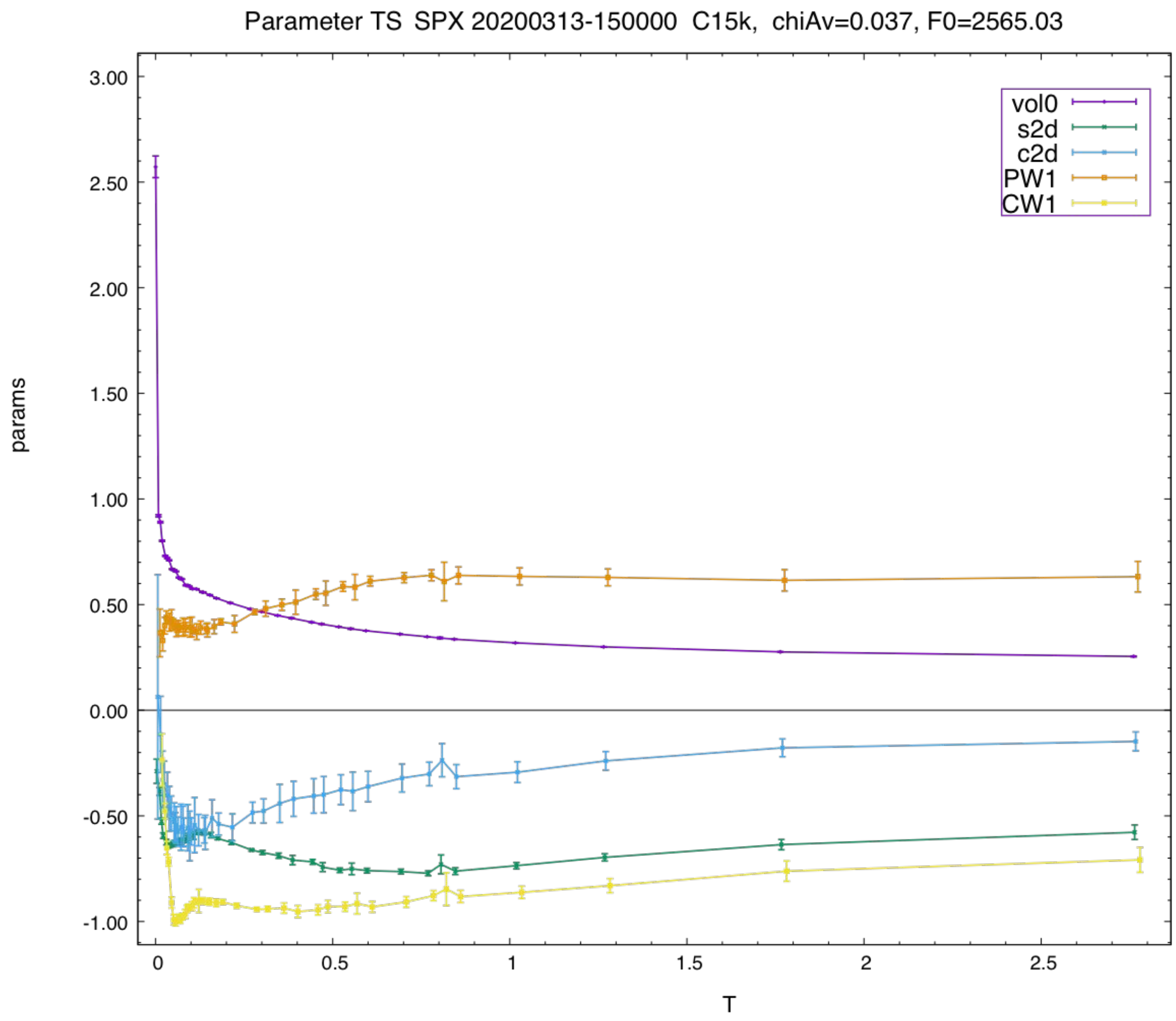
SPX 20200218 15:00

C15PM T = 3d



SPX 20200218 15:00

C15PM T = 1m



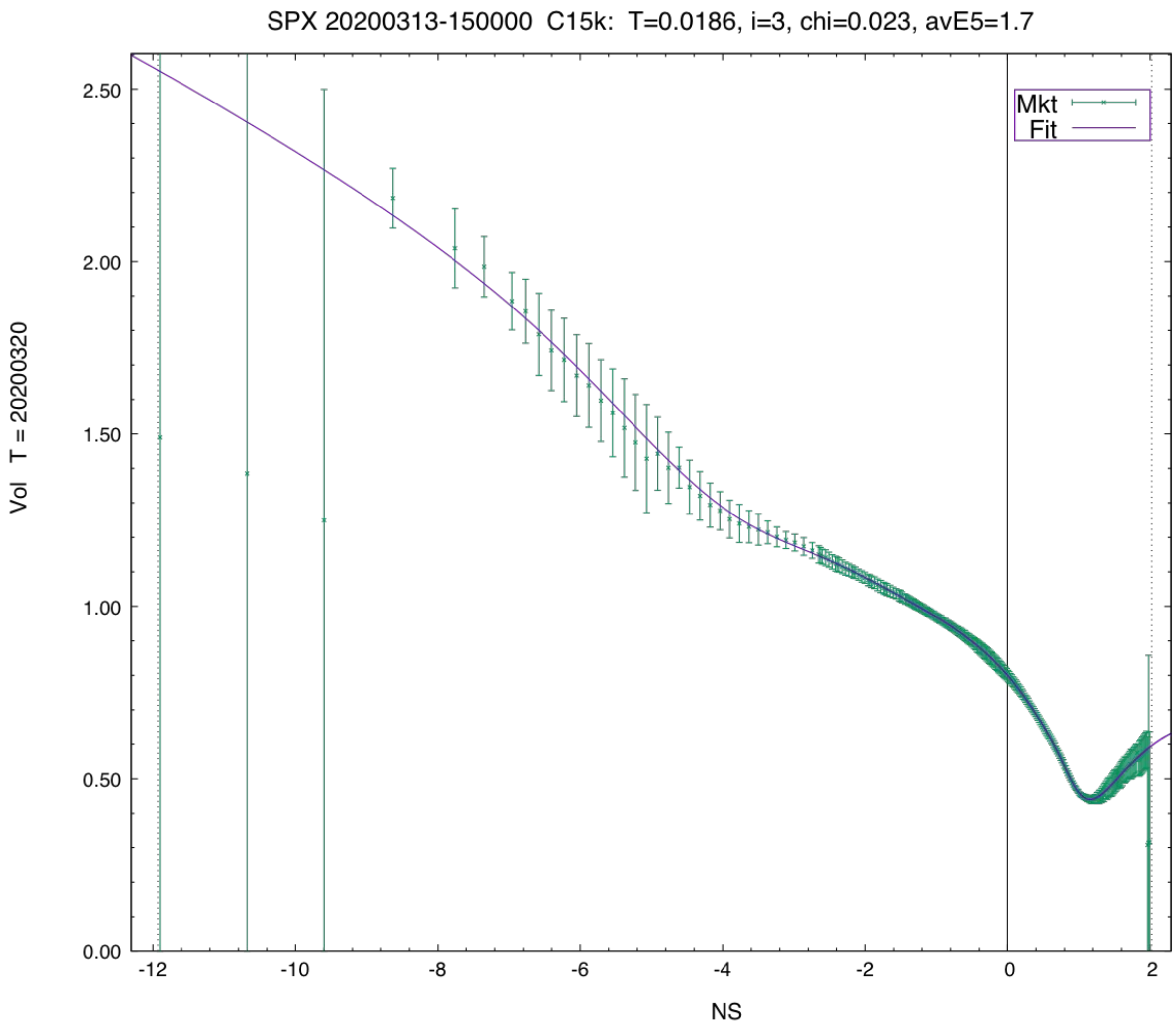
SPX 20200313 15:00

C15K Param Term-Structure during the **covid crash**

First 5 params…

All **c2** < 0 !!

Super-steep near call wing: **CW1**

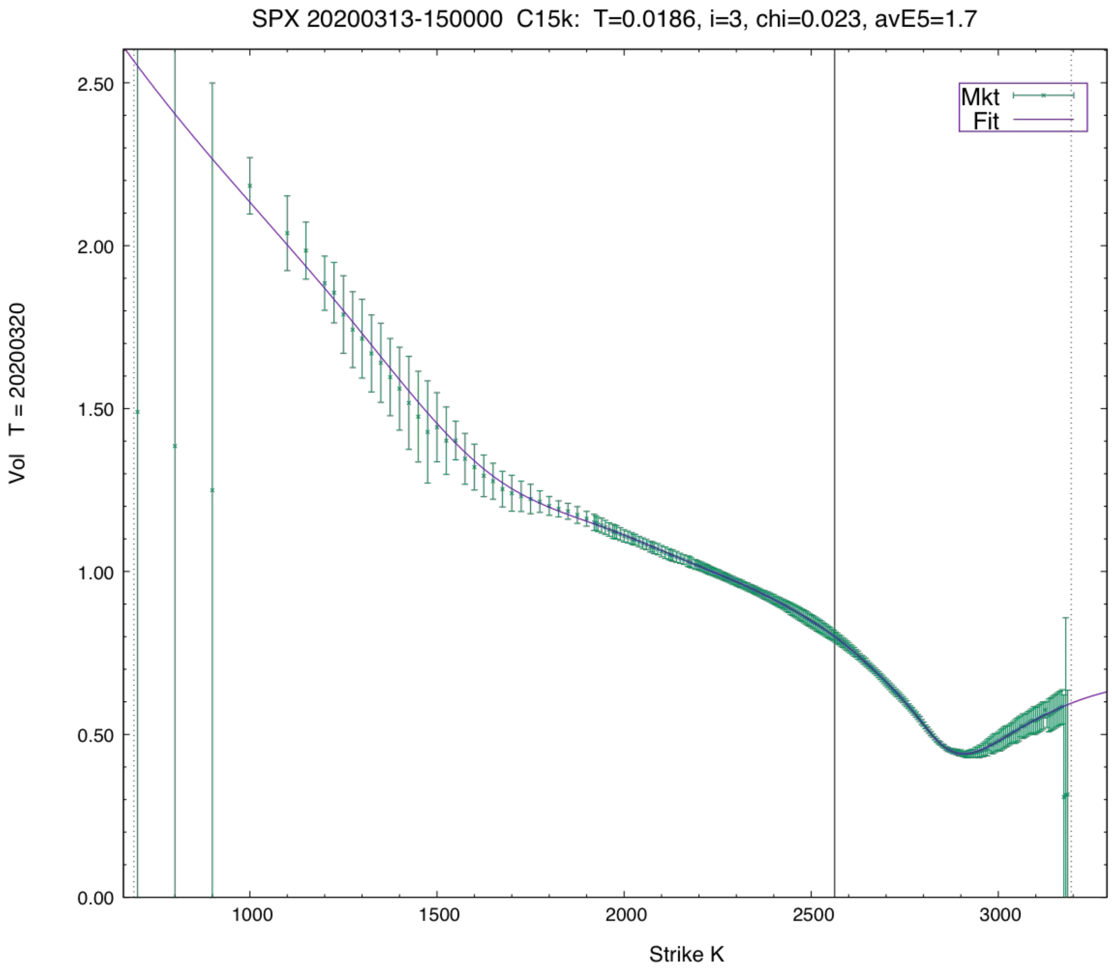


SPX 20200313 15:00

C15K T = 1w, in NS-space

Very compressed CW.

If fit followed PW more closely there would be fly arb…



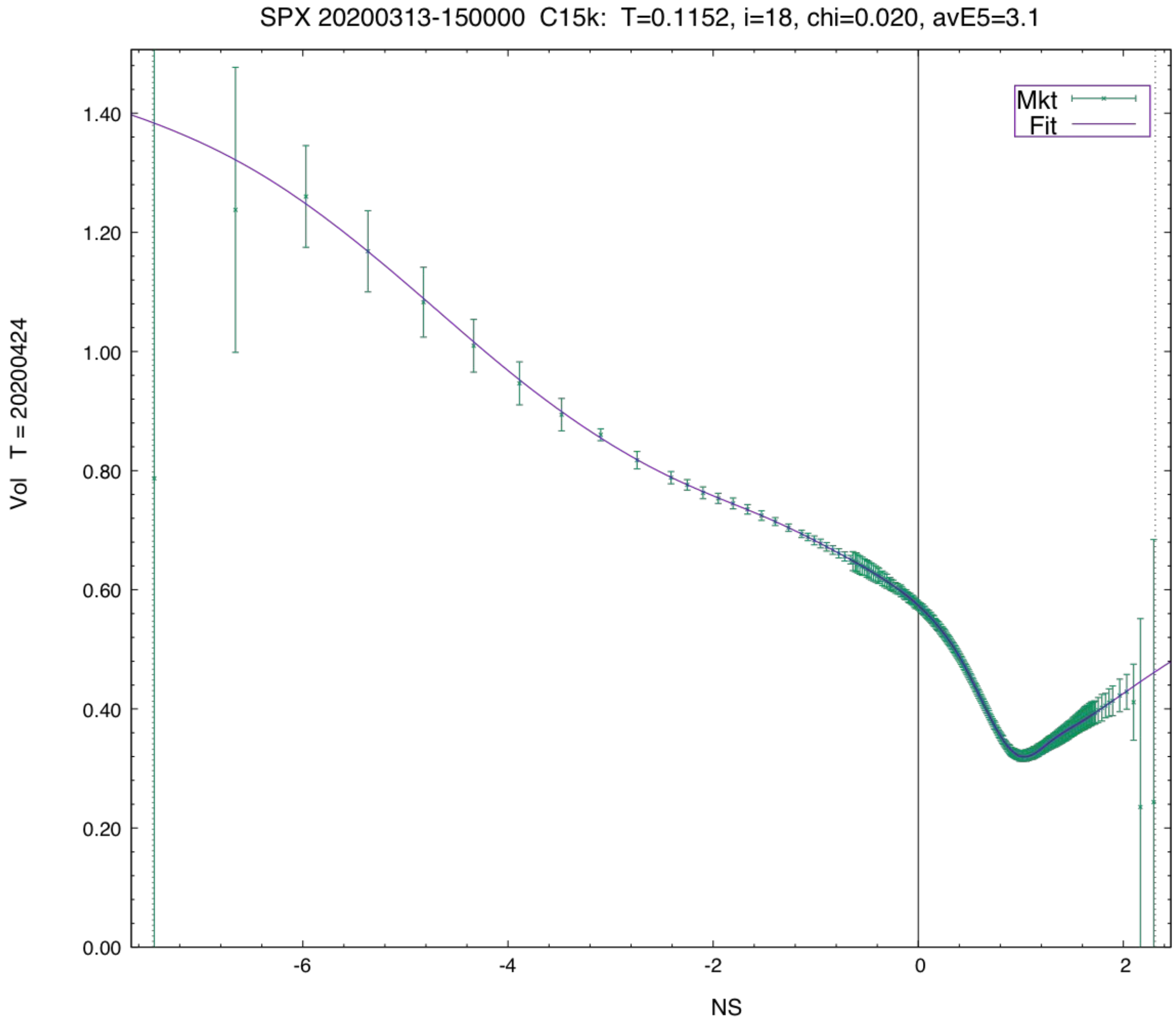
SPX 20200313 15:00

C15K T = 1w, in K-space

Very compressed CW.

If fit followed PW more closely there would be fly arb…

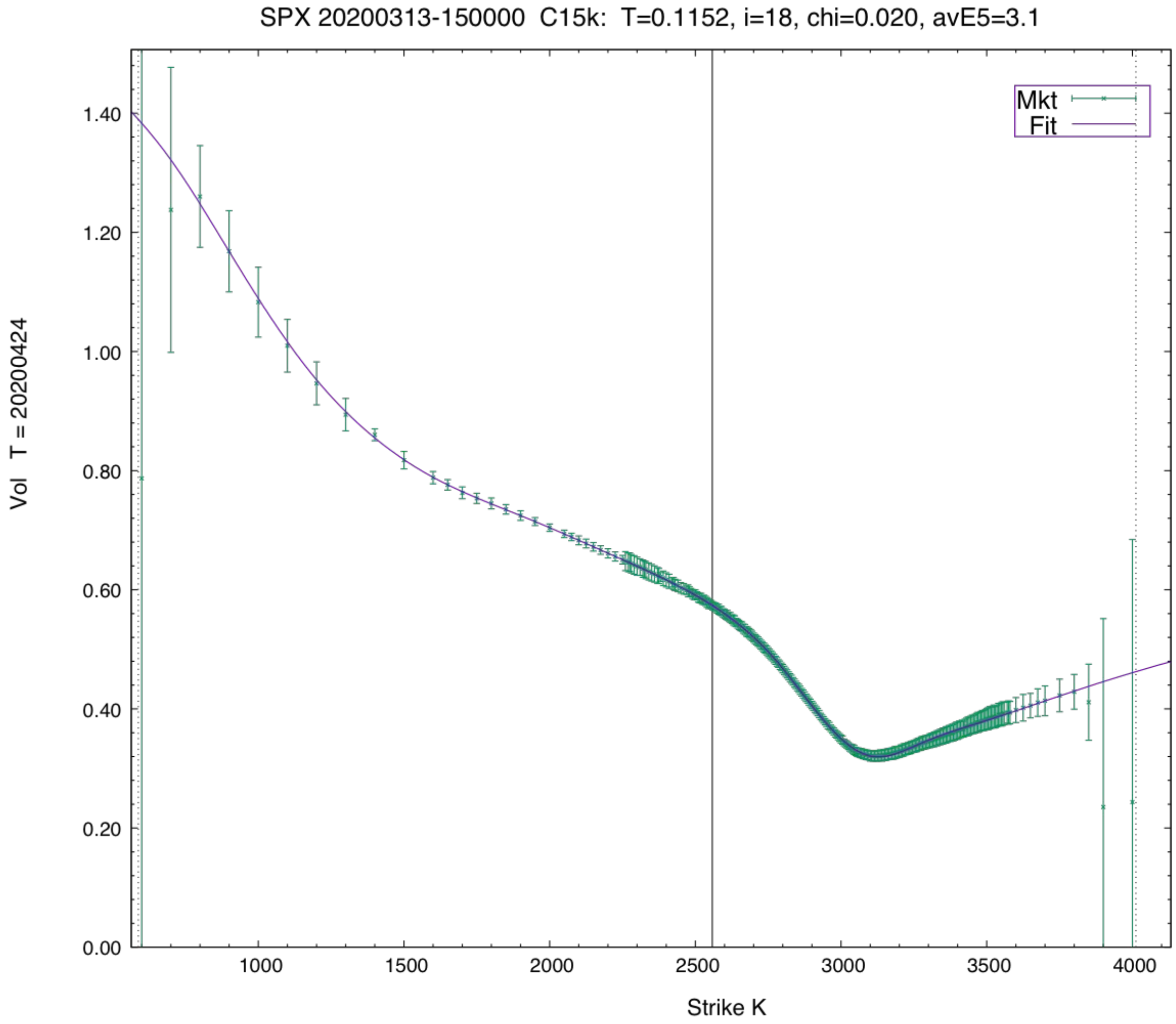
(Pretty well-functioning market over nK=379 strikes here…)



SPX 20200313 15:00

C15K T = 6w, in NS-space

Very compressed CW, very sharp knee…

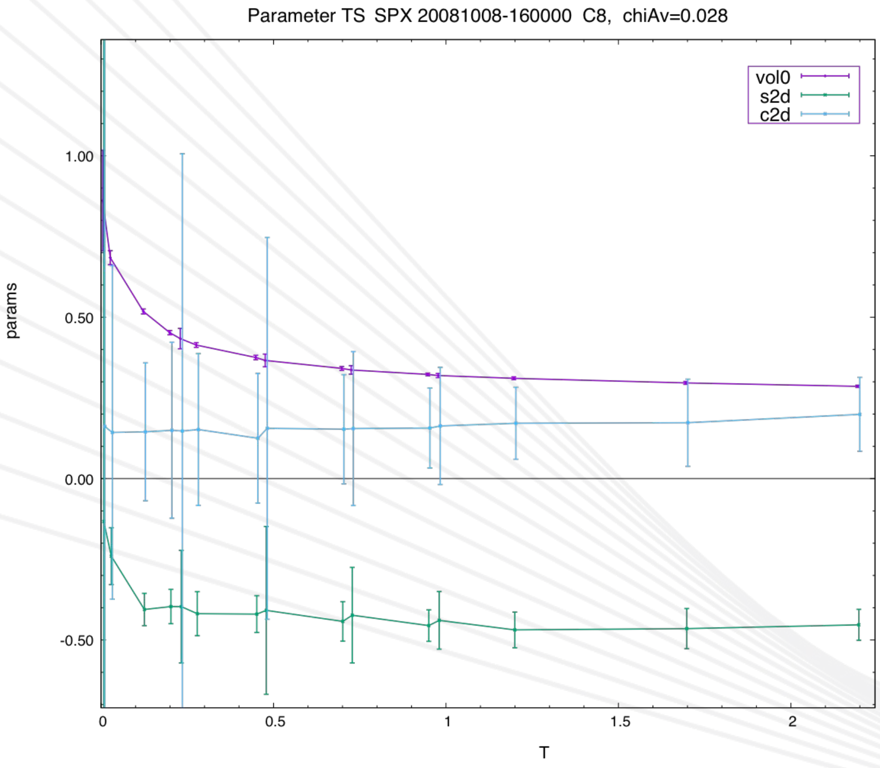


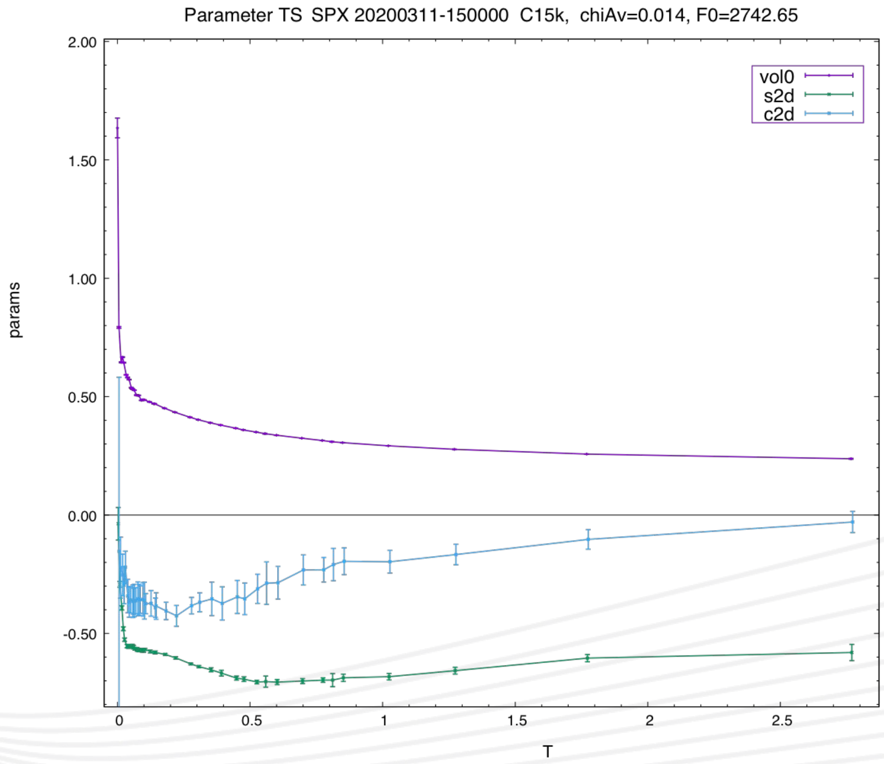
SPX 20200313 15:00

C15K T = 6w, in K-space

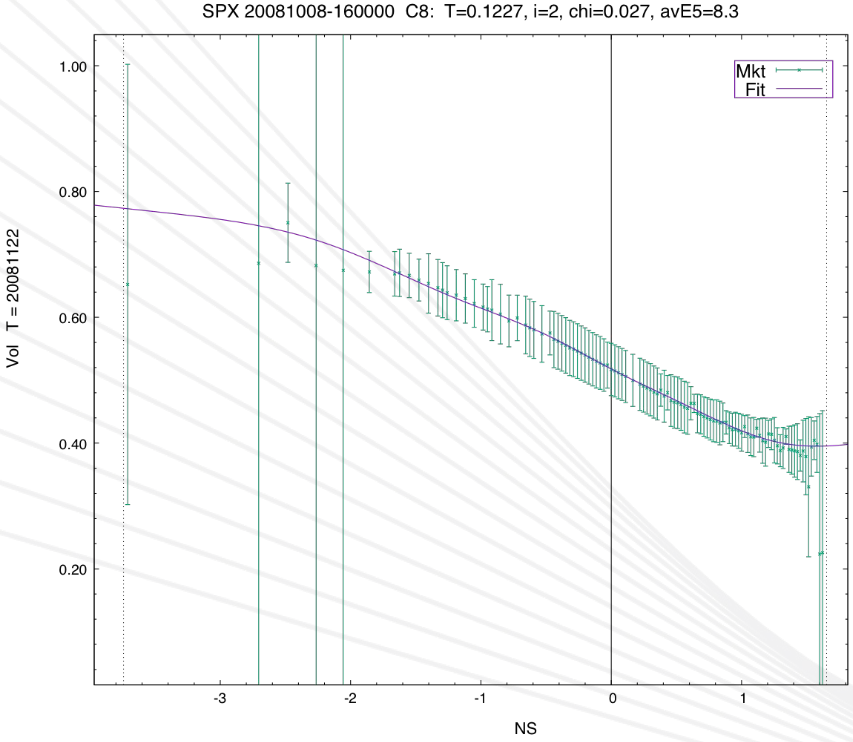
Very compressed CW, very sharp knee…

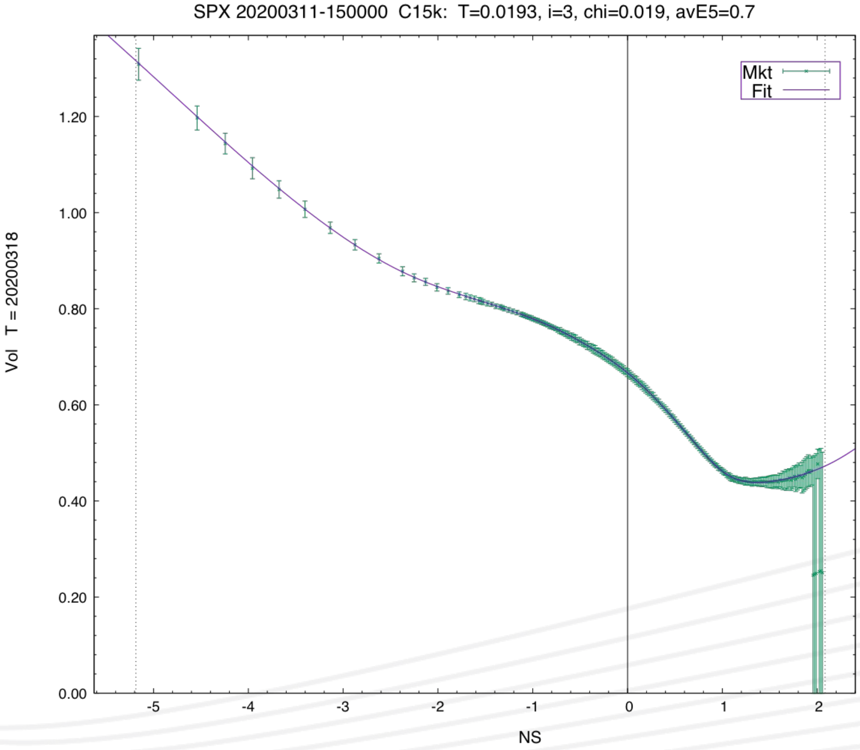
Parameter TS: 2008 versus 2020





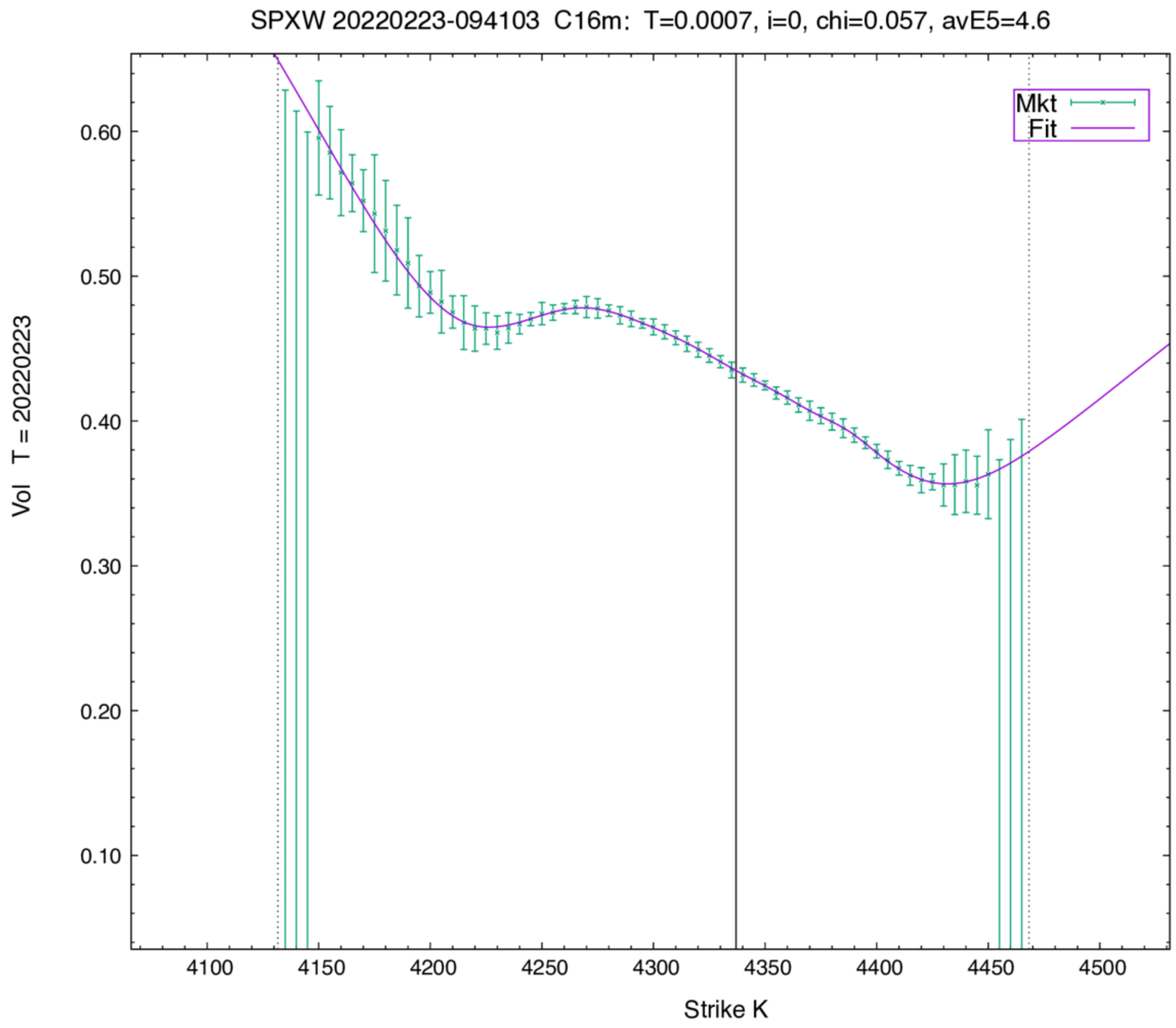
Vol Skews: 2008 versus 2020





Putin’s Put Wing

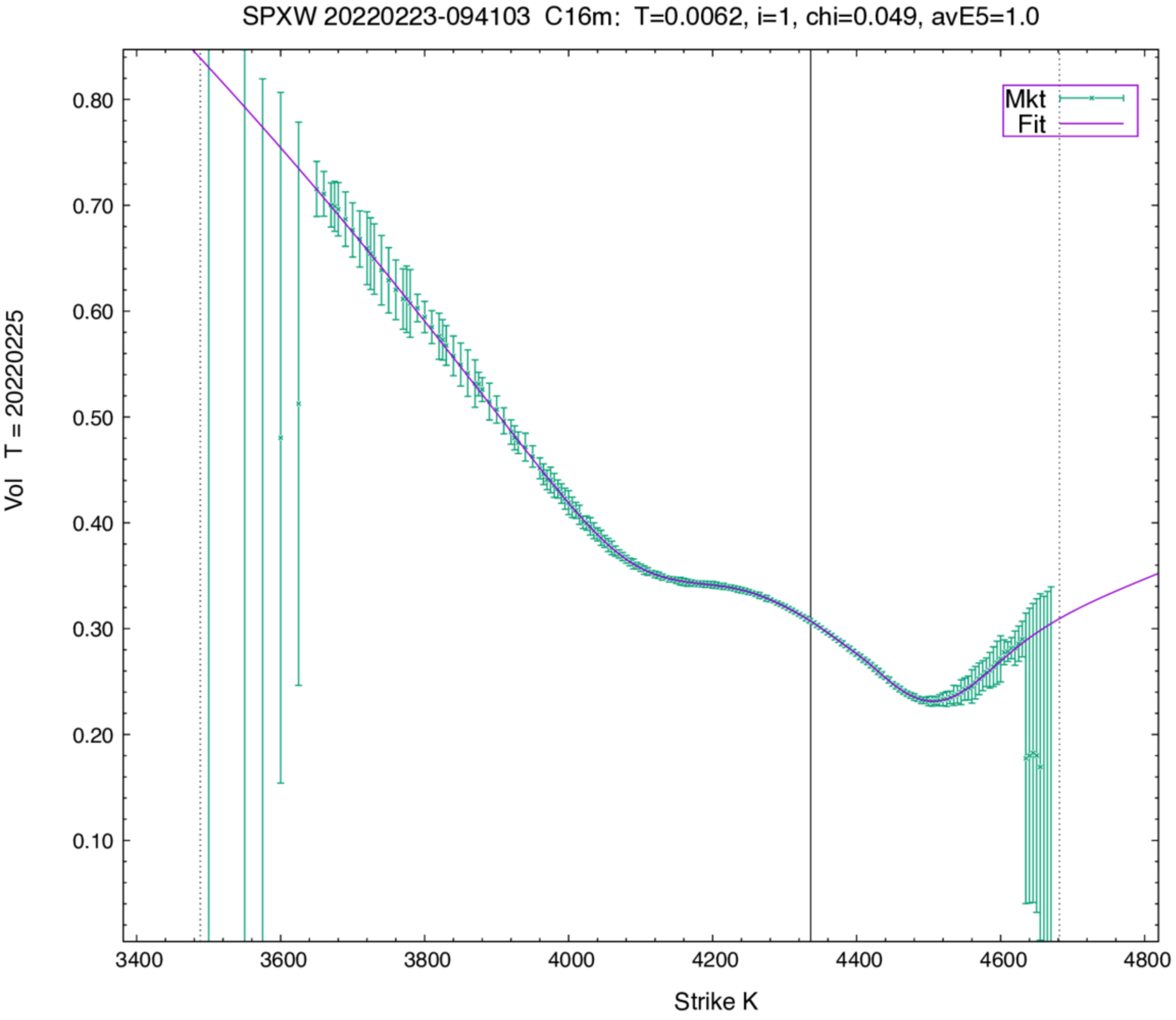
- On 2022-02-23, on the eve of the Ukraine invasion, the SPX daily expiry vol curve exhibited a peculiar, never-before-seen shape in the put wing.
- It lasted about 30 minutes, then disappeared, and the first expiry looked more like later expiries for the rest of the day.
- What does it mean?



SPX 20220223 9:41:03

T < 1d, in K-space

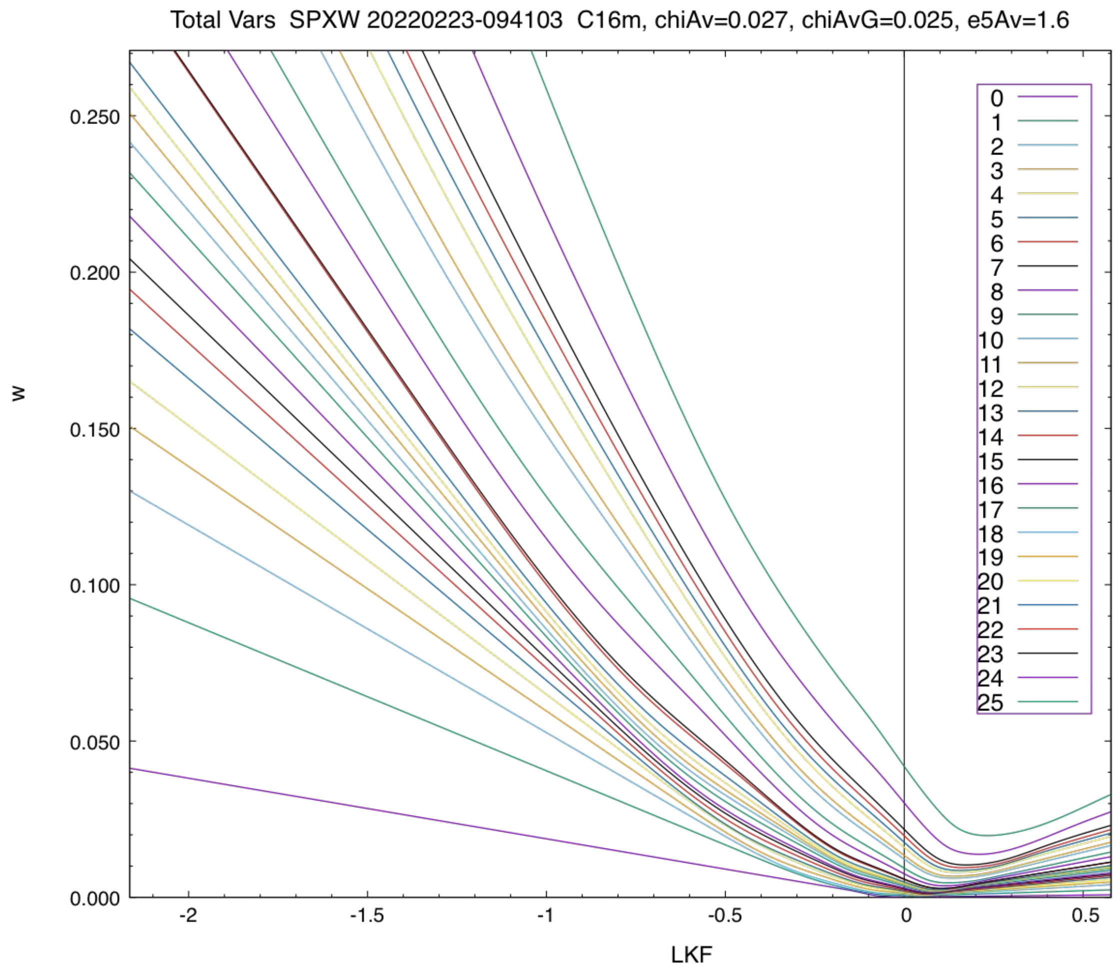
Putin's put wing – shape never seen before!



SPX 20220223 9:41:03

T < 3d, in K-space

The next expiry…



SPX 2022-02-23

Day before Ukraine invasion

C16m total variance plot

No crossings! (even i=14,15)

No calendar arb!

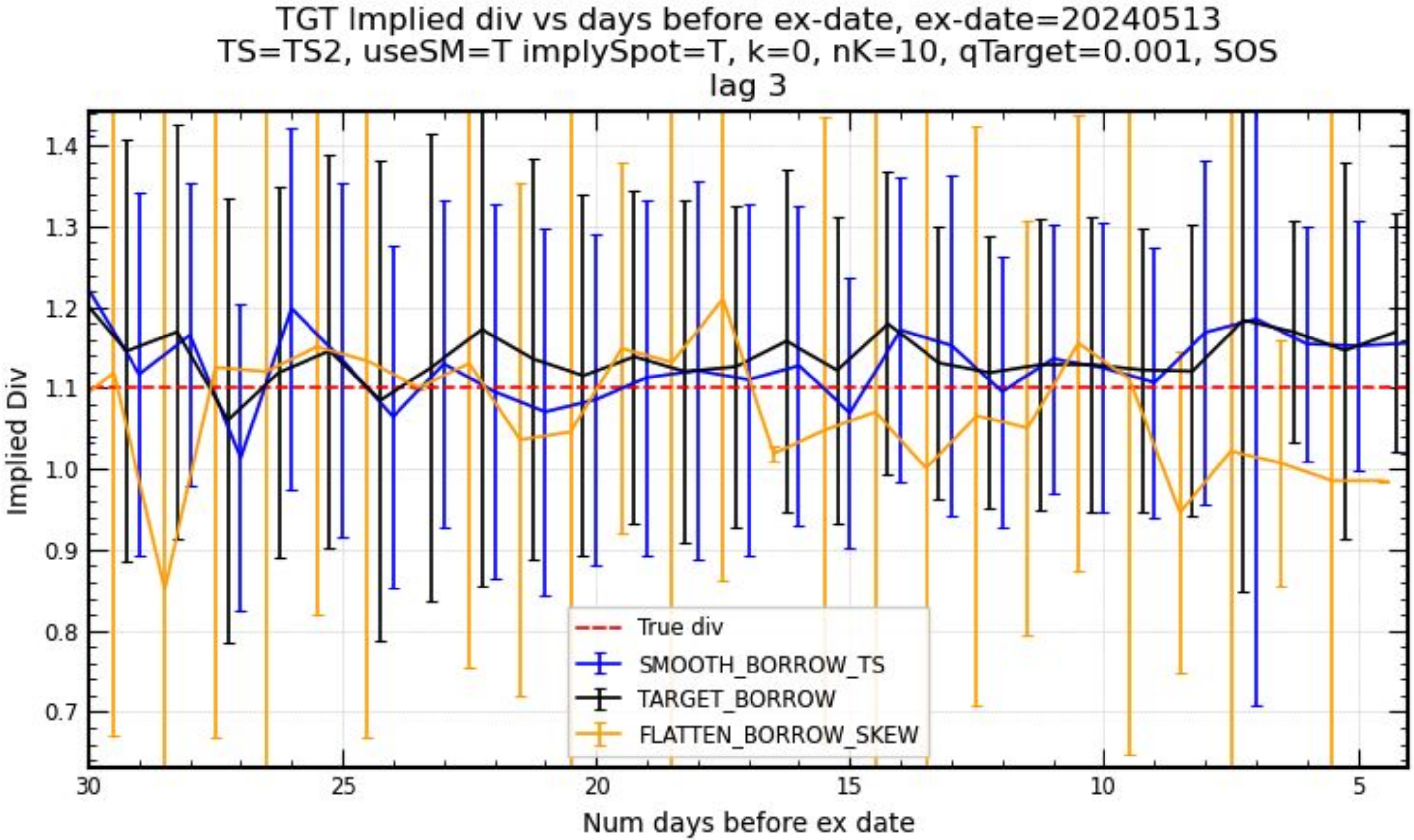
Just SPXW for clarity (and harder…)

Implying Dividends

- In practice, one often doesn’t know the upcoming dividend amount or date, nor the borrow cost.
  - And sometimes the available spot price is not synchronous with the options!
- With (arbitrarily accurate) synthetic data, it is not hard to disentangle the different effects that borrows and cash dividends (or a wrong spot) have.
- In the real world, with noisy data, it’s a much harder problem!
- To imply dividends:
  - If we know the borrow (and spot), it’s *relatively* easy…
  - Eliminate “kinks” in the borrow term-structure.
  - Eliminate “borrow skew” (different implied borrows for different strikes).

TGT implied dividend, ex-date = 20240513

Realized dividend vs three different impliedDiv methods



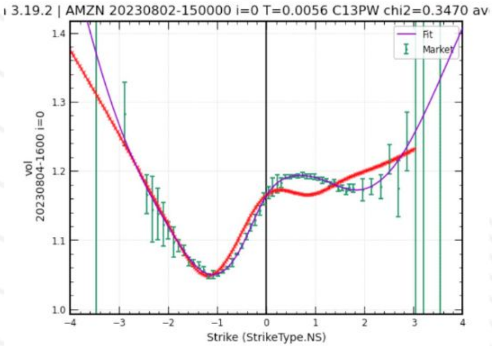
Event Modeling

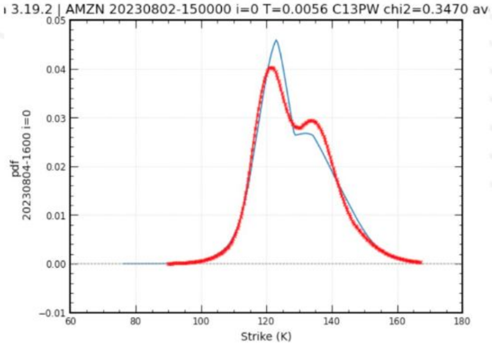
- Modeling an event like earnings properly requires a jump model.
  - With events at specific, non-random time(s).
- Minimal proper model: Two “**Merton Event Jumps**”, on top of a diffusive process, described by a “clean” aka “background” vol surface.
  - Various assumptions are possible about how to combine the clean surface with jumps.
  - Pricing: sum of Black formulas with an integral over jump sizes.
  - This is a proper model, unlike just describing the pdf of some expiry with a sum of log-normals, aka the Log-Normal Mixture “Model”.
  - So, one can ask if different expiries **consistent with the same jumps**!

AMZN 2023-08-02 earnings

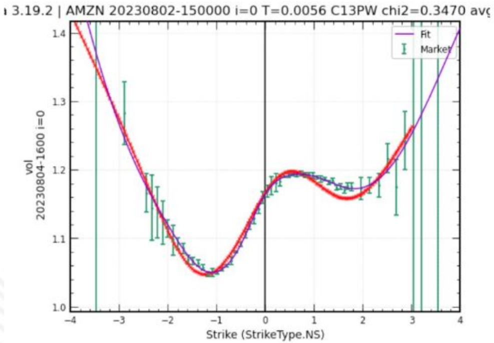
Calibrating three 2-jump models, with 2,3,4 parameters:

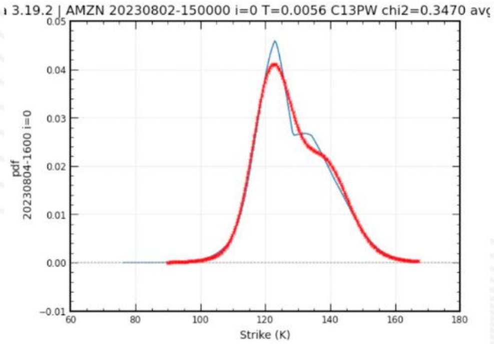
ASYMMETRIC_DISCRETE_JUMP
J= [1.0712686447685513, 0.9549850455121303]
p= [0.387113529128091l, 0.6128864708719088]
s= [0.0, 0.0]



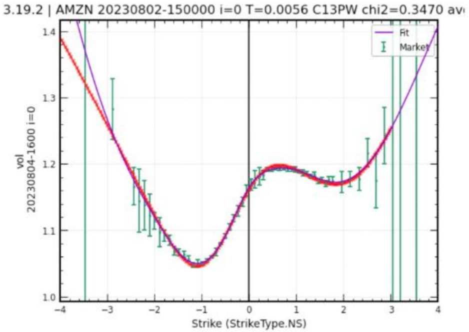


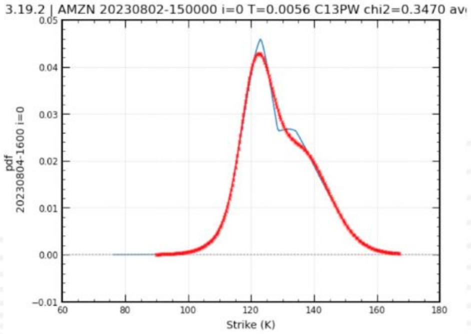
ASYMMETRIC_MERTON_JUMP
J= [0.9555676988524993, 1.0839264422585266]
p= [0.6538428160912148, 0.34615718390878525]
s= [0.044851664518220605, 0.044851664518220605]





ASYMMETRIC_MERTON_JUMPS
J= [1.0594025258161424, 0.9441995069221296]
p= [0.48436658703542335, 0.5156334129645767]
s= [0.0588065555709617, 0.03688014214112789]

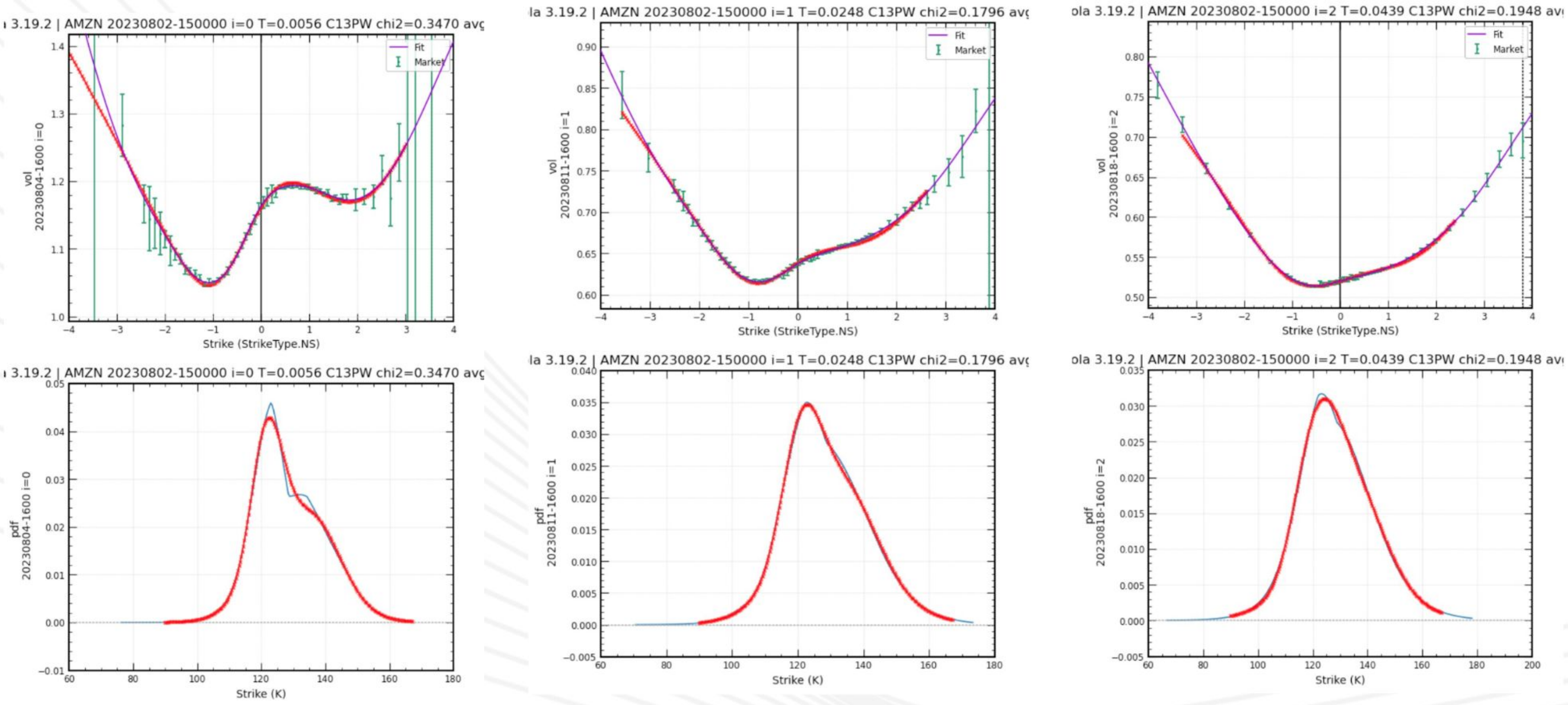




nAfter=3
vctC = C5

AMZN 2023-08-02 earnings

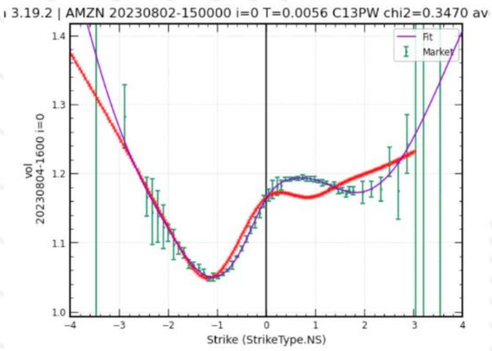
The best 2-jump model works for three expiries after earnings:

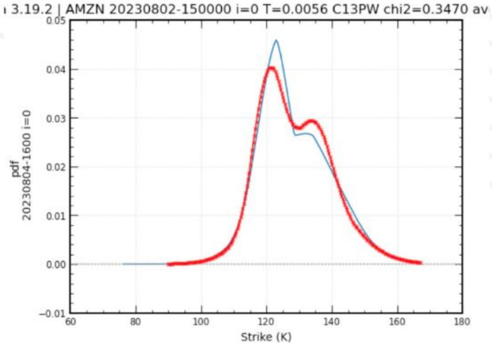


AMZN 2023-08-02 earnings

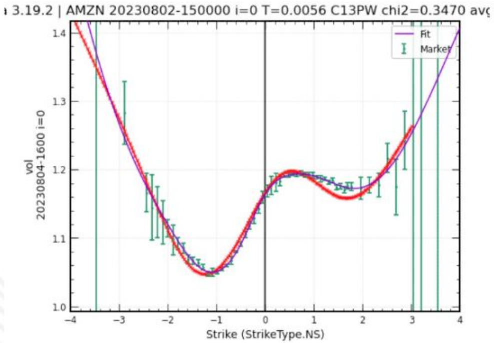
Calibrating three 2-jump models, with 2,3,4 parameters:

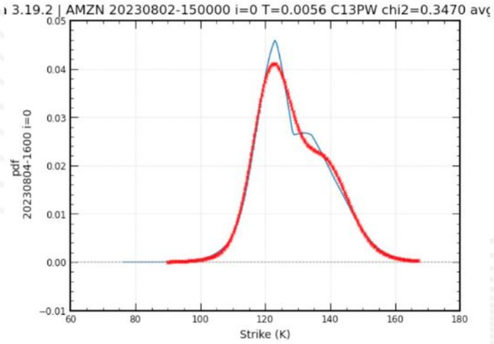
ASYMMETRIC_DISCRETE_JUMP
J= [1.071268644768551 3, 0.9549850455121303]
p= [0.387113529128091 1, 0.6128864708719088]
s= [0.0, 0.0]



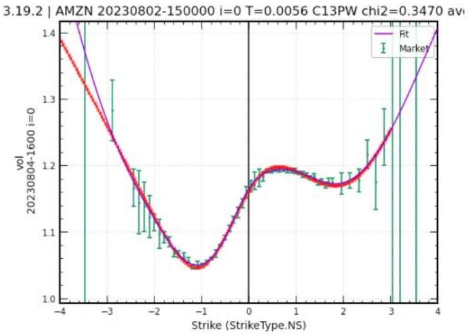


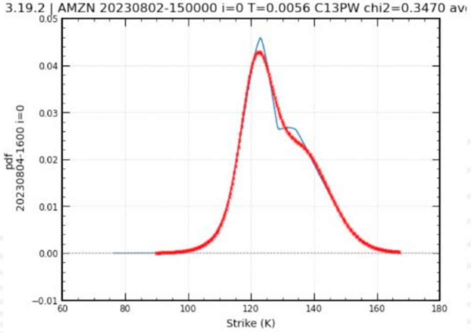
ASYMMETRIC_MERTON_JUMP
J= [0.9555676988524993, 1.0839264422585266]
p= [0.6538428160912148, 0.34615718390878525]
s= [0.044851664518220605, 0.044851664518220605]





ASYMMETRIC_MERTON_JUMPS
J= [1.0594025258161424, 0.9441995069221296]
p= [0.48436658703542335, 0.5156334129645767]
s= [0.0588065555709617, 0.03688014214112789]

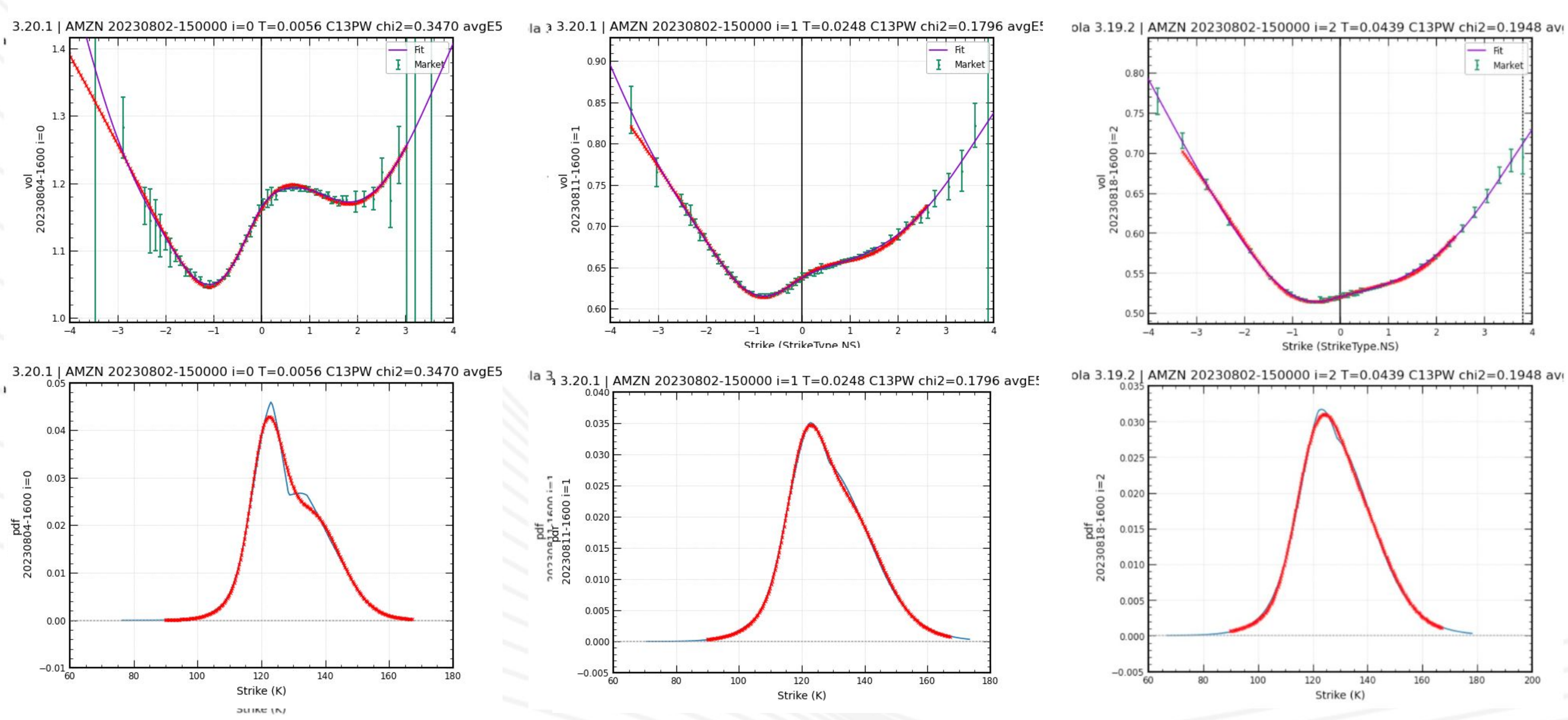




nAfter=3
vctC = C5

AMZN 2023-08-02 earnings

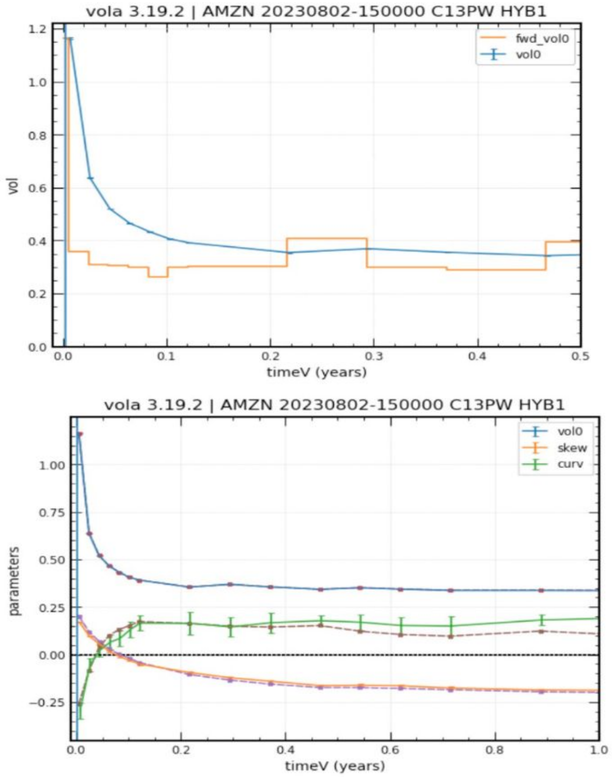
The best 2-jump model works for three expiries after earnings:



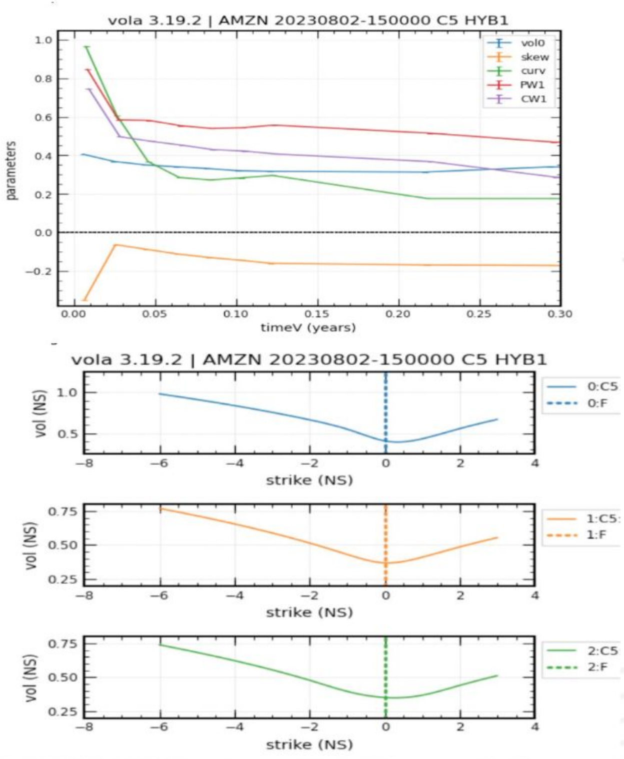
AMZN 2023-08-02 earnings

Though the clean VS is not quite as clean as we would like:

Dirty



Clean

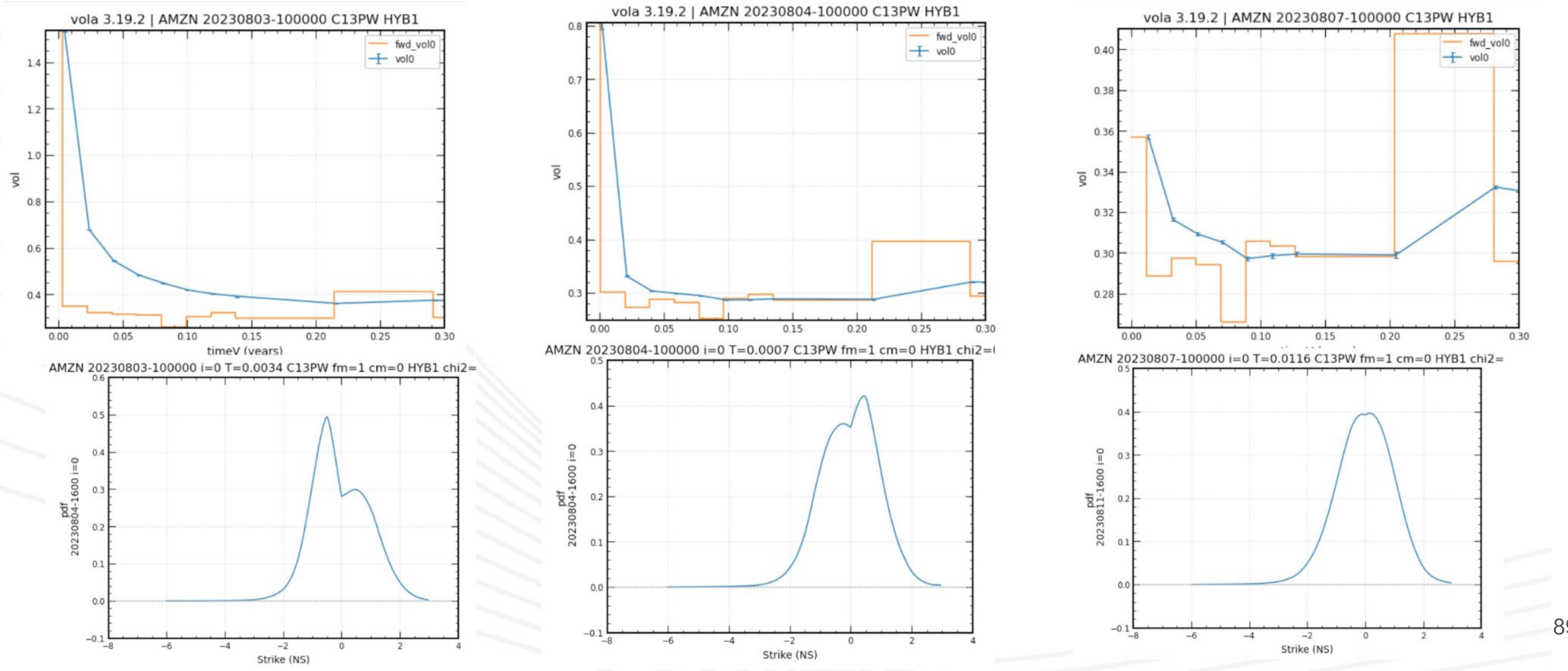


nAfter=3
vctC = C5
MertonJumps

```
J= [1.0594025258161424, 0.9441995069221296]
p= [0.48436658703542335, 0.5156334129645767]
s= [0.0588065555709617, 0.03688014214112789]
```

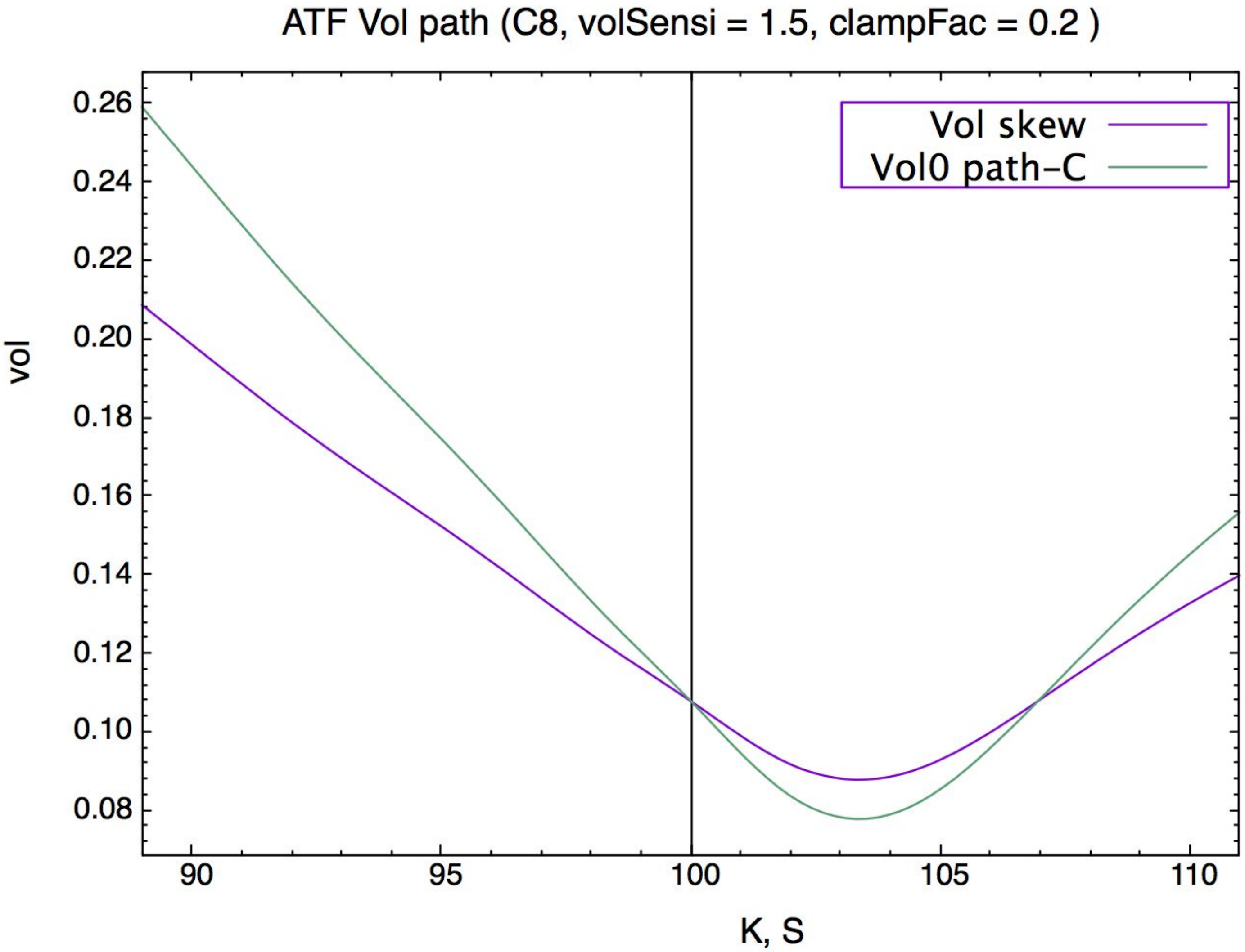
AMZN 2023-08-02 earnings

The event is not quite discrete in time, there are “aftershocks”:



Spot-Vol Dynamics: Basics

- **Shape** (by NS or Δ) is much more **stable** than overall vol level (vol0 aka ATF vol).
- **ATF vol dynamics** is very well described by one dimensionless number, **SSR** aka vol sensitivity aka super-skew, which is the ratio of vol0-path & skew slopes.
- Very **simple dynamics** in terms of NS **vol parameters** (e.g. just ATF vol), gives **complicated vol-by-strike dynamics**, that actually describes market moves.
  - It also gives the correct adjusted (aka smart aka **skew**) **delta** and gamma (see LinkedIn article).
- We will illustrate each of these points.



Spot-Vol Dynamics

ATF “vol path”

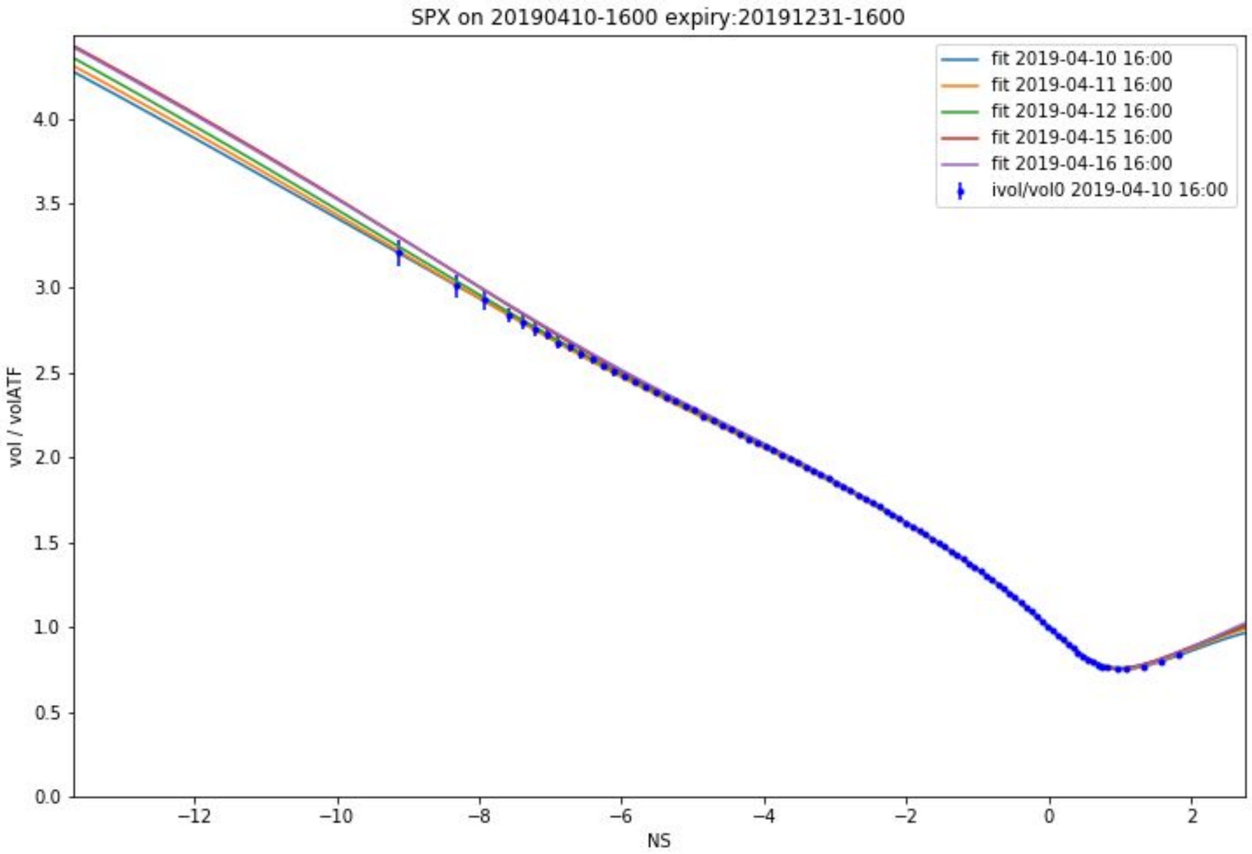
SSR = 1.5

“Along curve”

No clamps

SPX Spot-Vol Dynamics: Then and Now

- In the olden days:
  - Virtually no shape dynamics.
  - Overall vol level dynamics described very well by one SSR with little term-structure (TS).
  - 1 < SSR < 2, with 2 reached only on big down days. Typical value SSR=1.3.
- Nowadays:
  - There is often **term-structure** in SSR, with SSR(T > 1y) closer to typical values.
  - There is occasionally, e.g. on some big down days, **shape dynamics**, eg in c2.
  - **SSR > 2 and SSR < 1** can happen, on short end.
  - Some horizon dependence (1min, 5min, etc), including intraday vs overnight differences.
  - More “fluctuations”, in **path-dependent** manner (cf. Guyon), around typical values.
  - Open Q: How strong is path-dependency effect relative to levels set by “SSR regime” ?

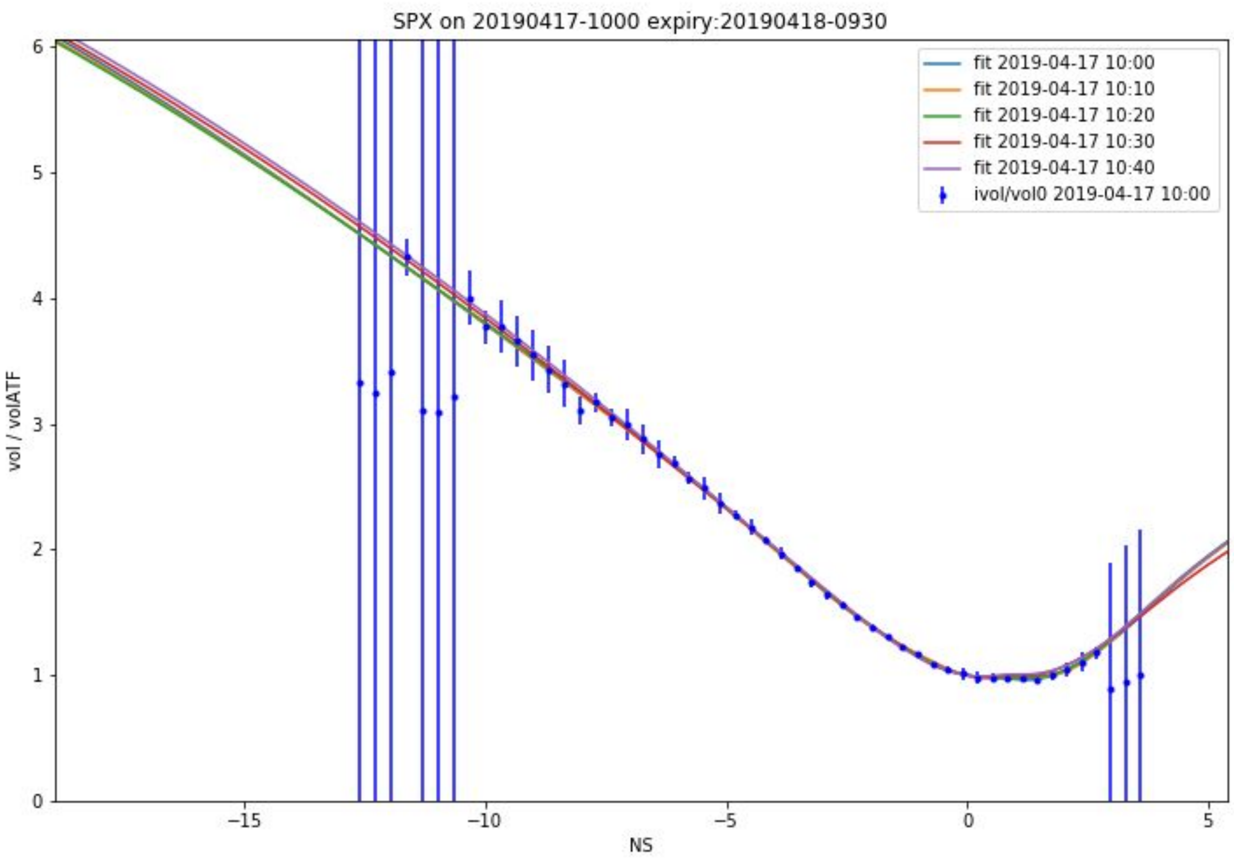


Stability of NS Shape

SPX 20190410 T = 9m

Shape stable over many days, while underlier moves around.

Also, no floppy wings!

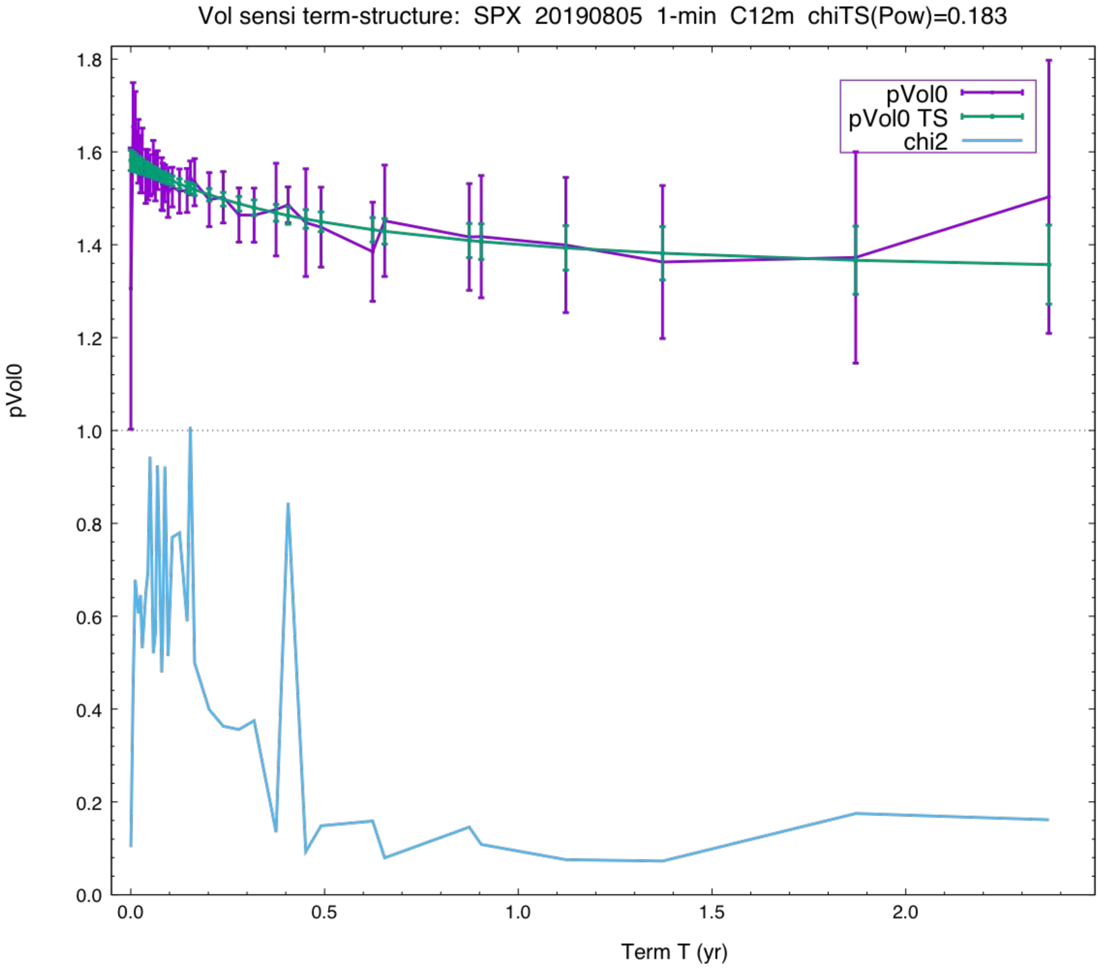


Stability of NS Shape

SPX 20190410 T = 1d

Shape stable even on last day

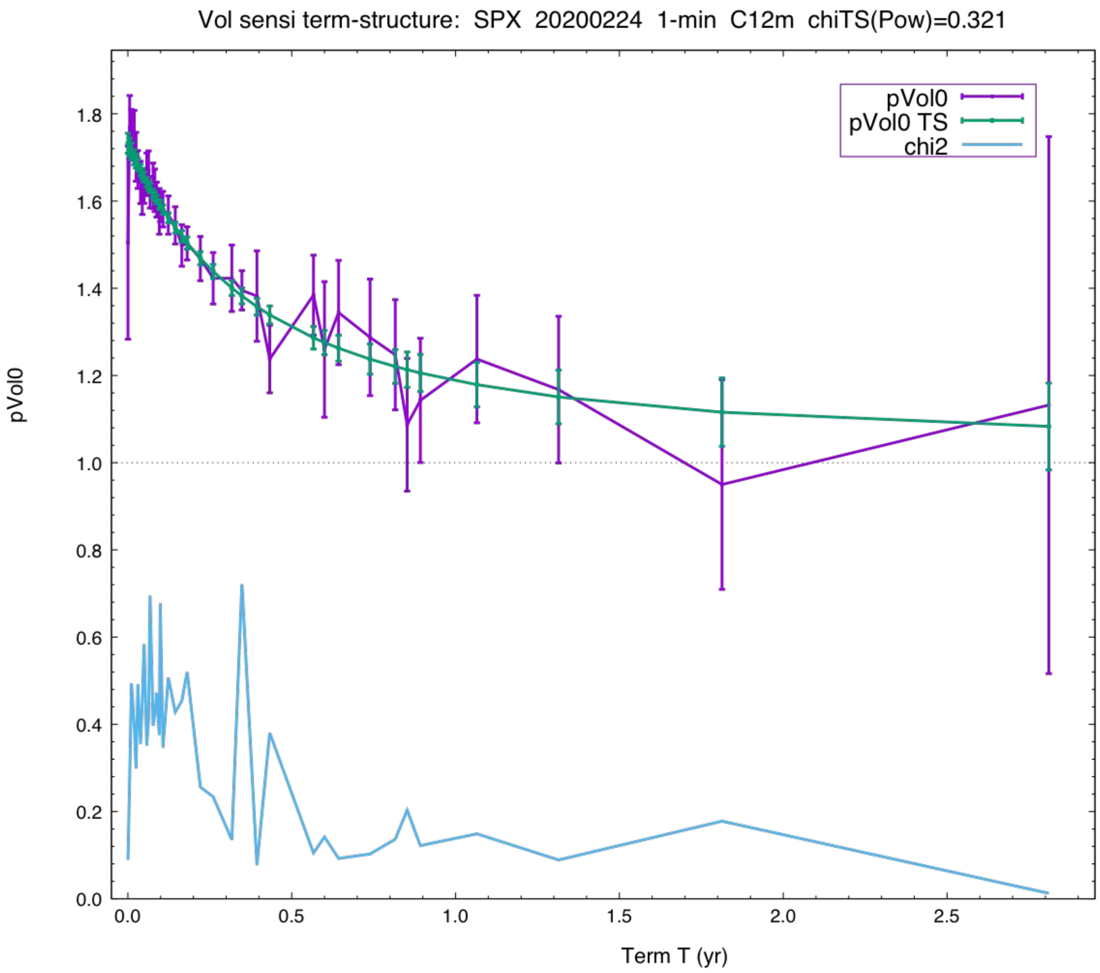
Also, no floppy wings!



SPX 20190805

Vol sensitivity (SSR) term-structure

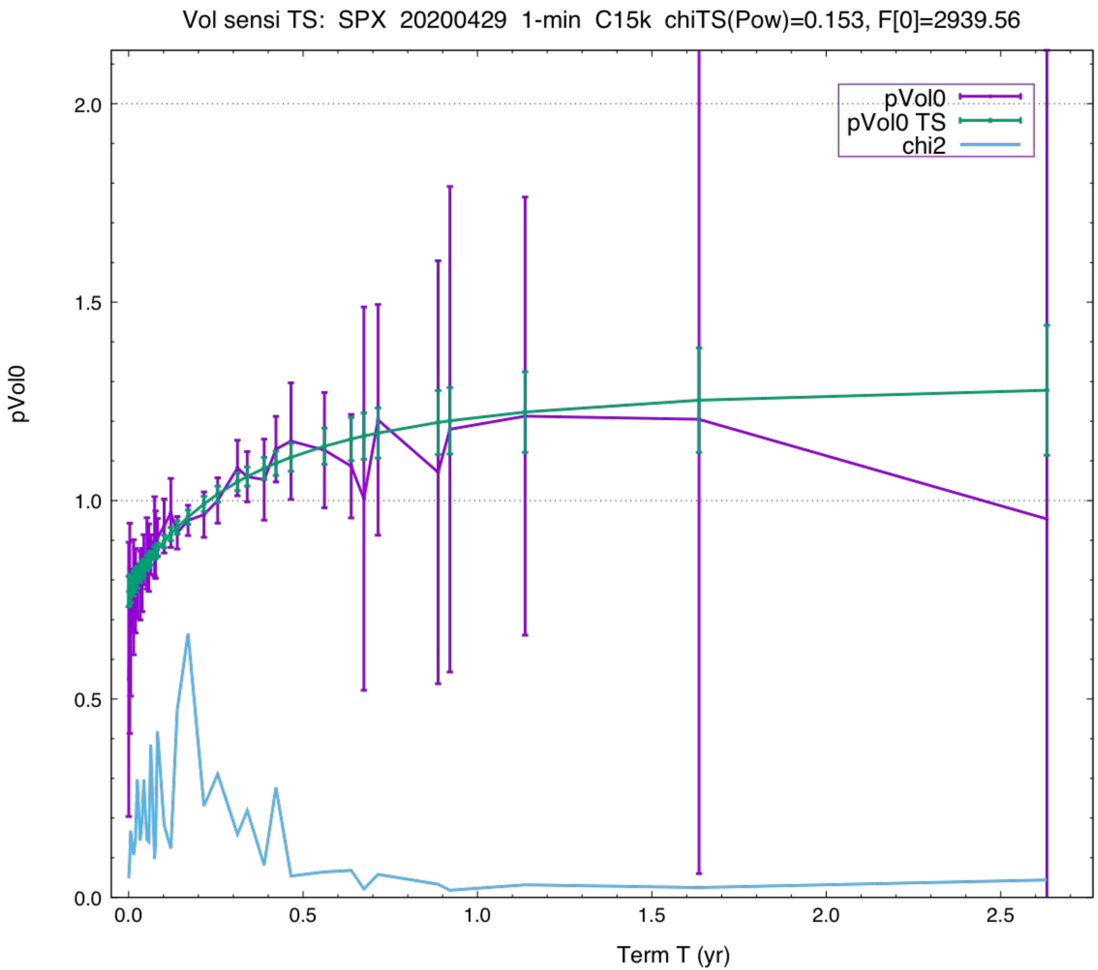
Parametric fit for robustness on small data sets (can be done intra-day)



SPX 20200224

Vol sensitivity (SSR) term-structure

Parametric fit for robustness on small data sets



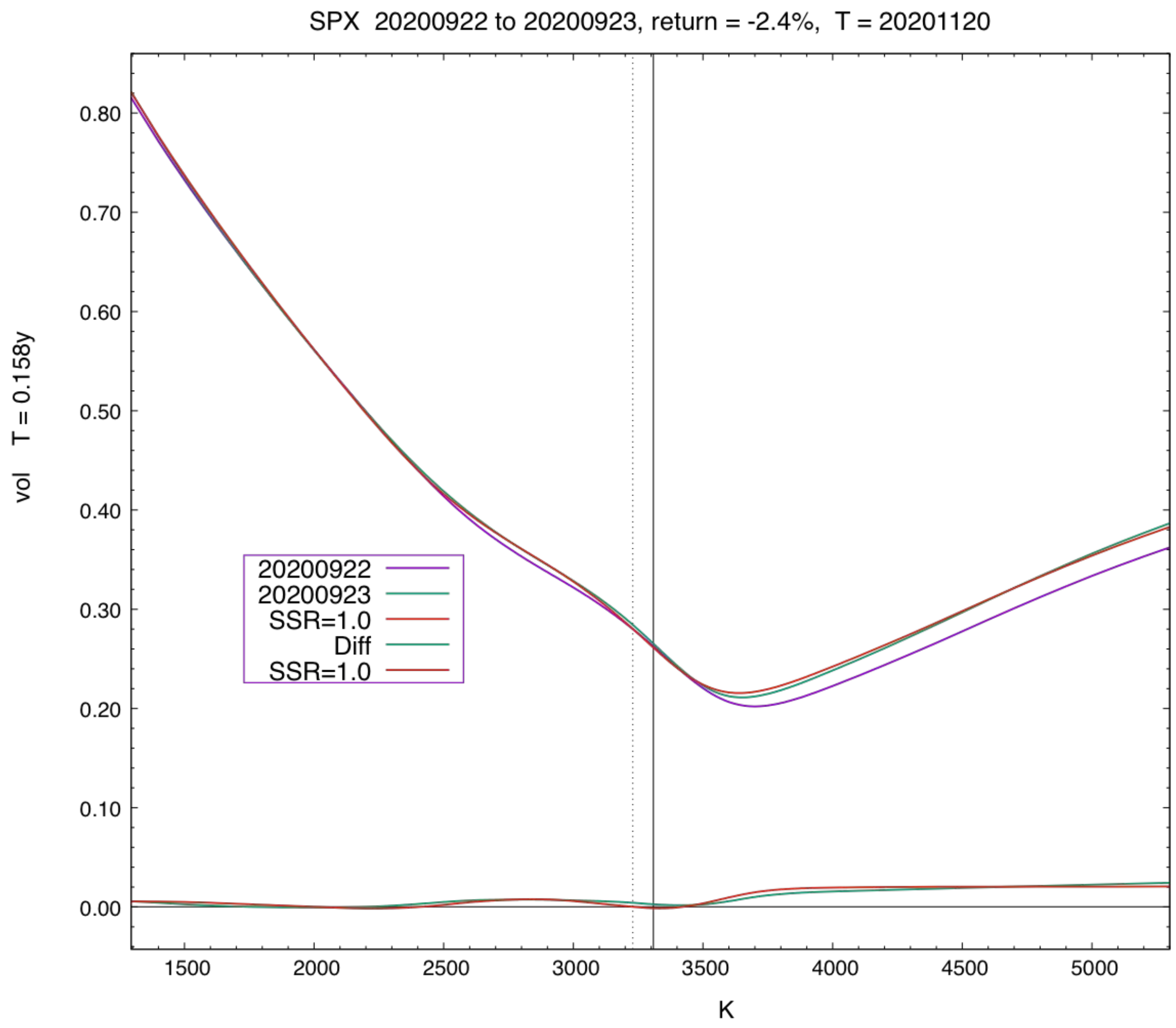
SPX 20200429

Vol sensitivity (SSR) term-structure

On up-days can be upward-sloping, and SSR < 1 at least for some terms

Spot-Vol Dynamics Myths

- **Myth: Common "vol regimes" are "sticky-by-strike" or "sticky-by-delta"**
  - Sticky-strike scenarios are still commonly used by risk departments.
- In equities, at least, **neither has happened for 20y+**.
- Sticky-delta implies SSR=0 for all terms. Never happens.
- Sticky-strike implies SSR=1 (i.e. sticky-strike around ATM).
  - Even when vols are sticky-by-strike around ATM, they never are in the wings.
  - There are many examples. Let's look at some.



Close-to-close spot vol dynamics

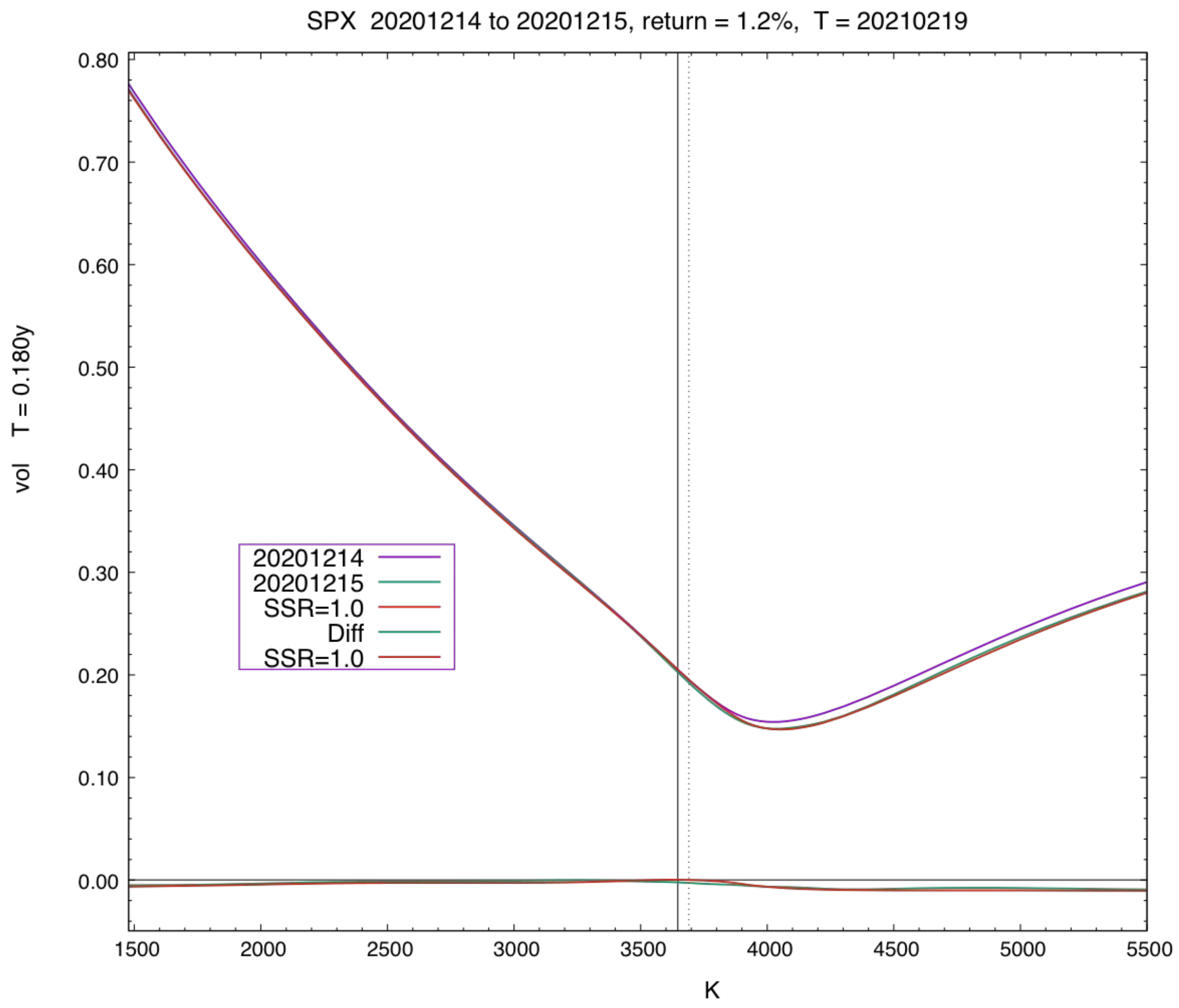
SPX 2020-09-22 to 2020-09-23

SSR=1, but NO sticky strike in the wings.

Instead: Shapes are sticky-by-NS!

Non-trivially so in the call and put wing!

This down-day comes after a sequence of (minor) down days, and SSR has mean-reverted to 1.0

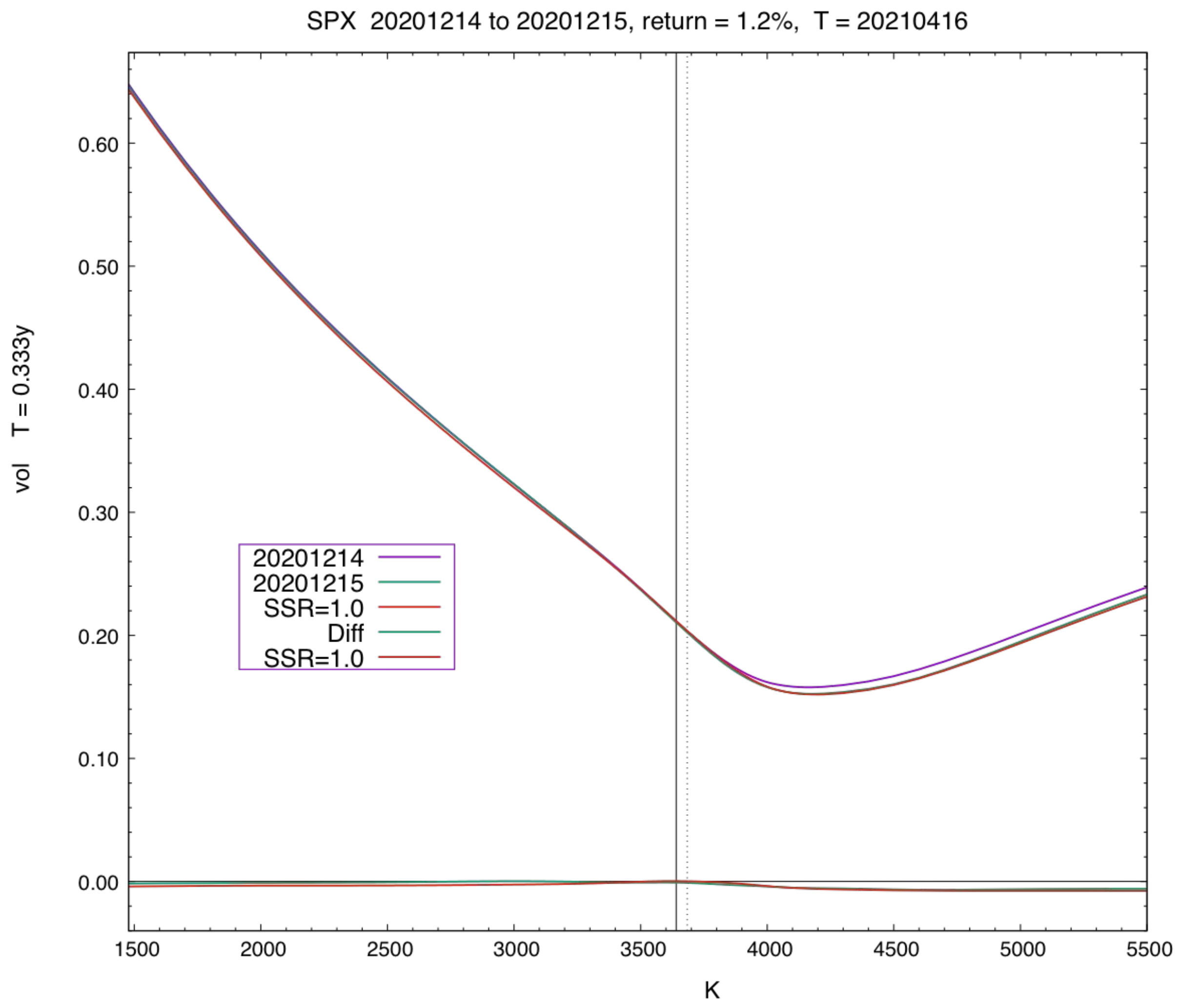


Close-to-close spot vol dynamics

SPX 2020-12-14 to 2020-12-15

T = 9w, SSR = 1

Fixed NS shape assumption works **amazingly** well!

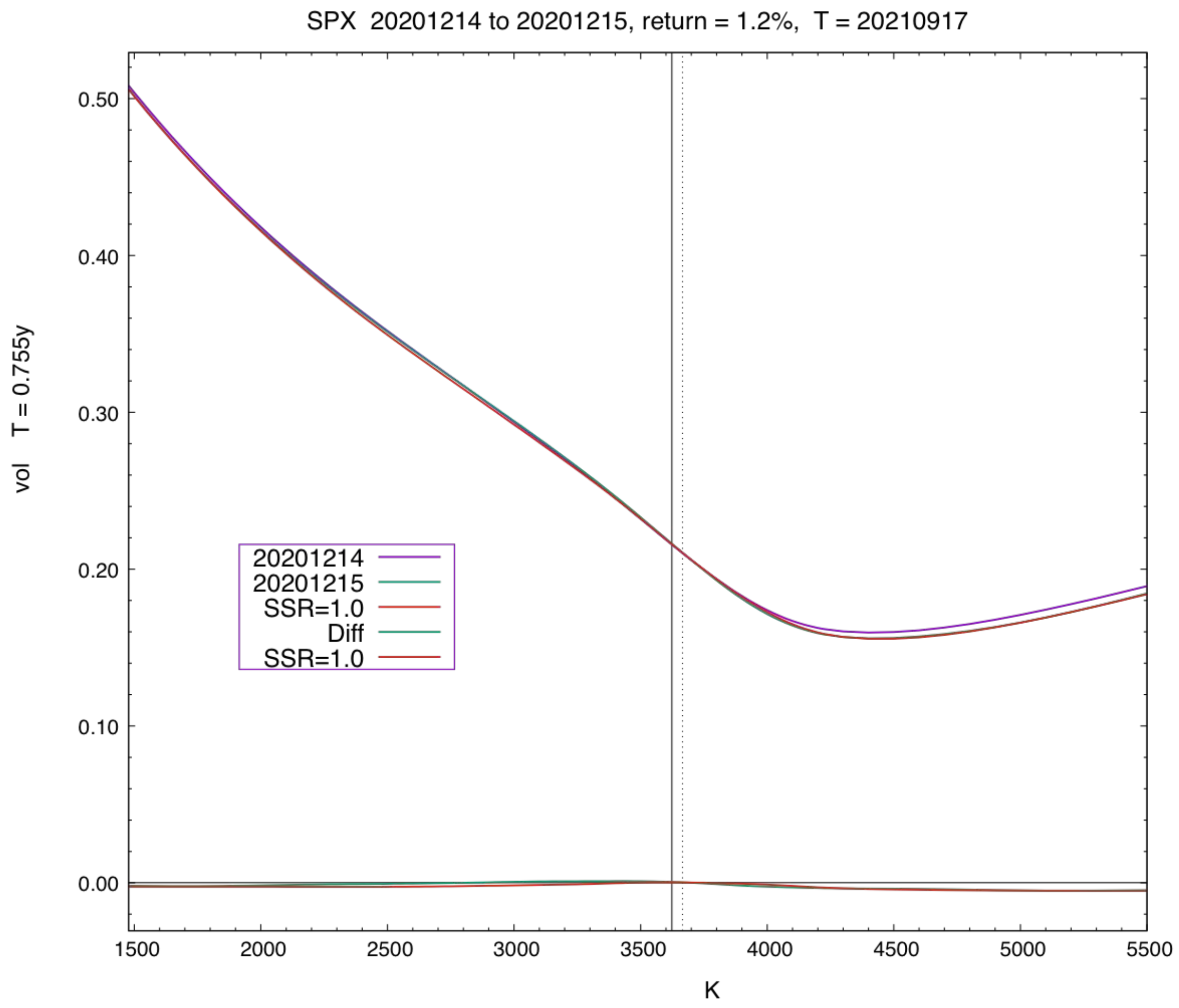


Close-to-close spot vol dynamics

SPX 2020-12-14 to 2020-12-15

T = 4m, SSR = 1

Fixed NS shape assumption works **amazingly** well!



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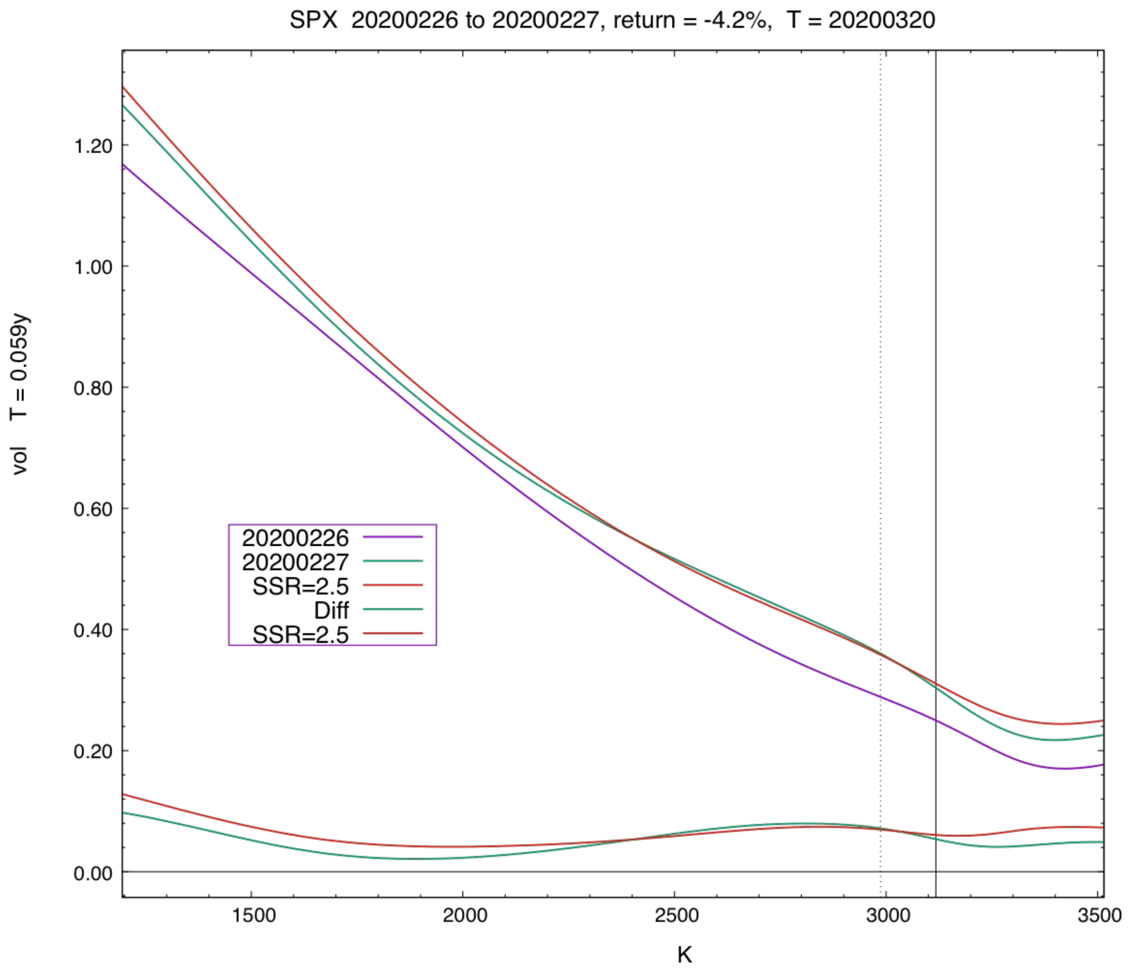


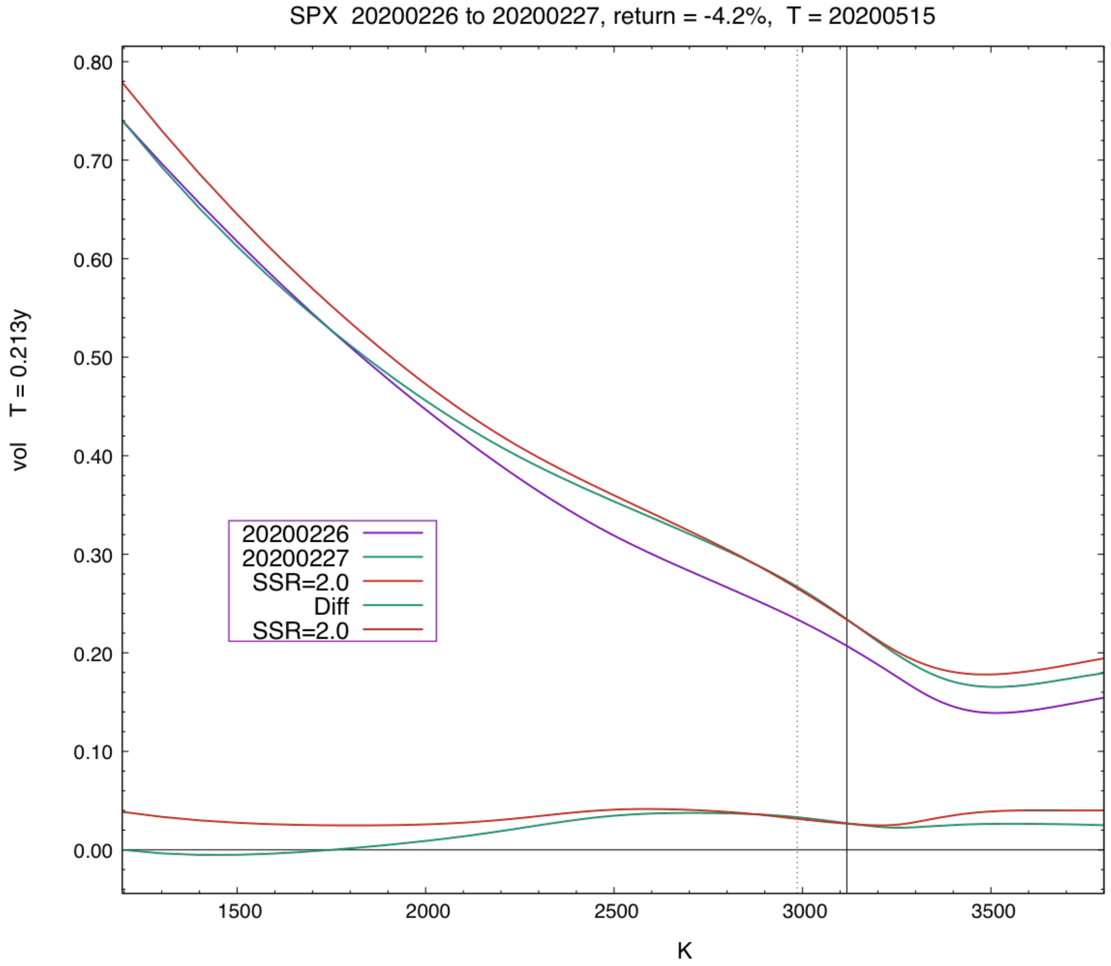
Close-to-close spot vol dynamics

SPX 2020-02-26 to 2020-02-27

Covid crash!

T = 3w, SSR = 2.5

Evidence for c2-spot-sensitivity > 0



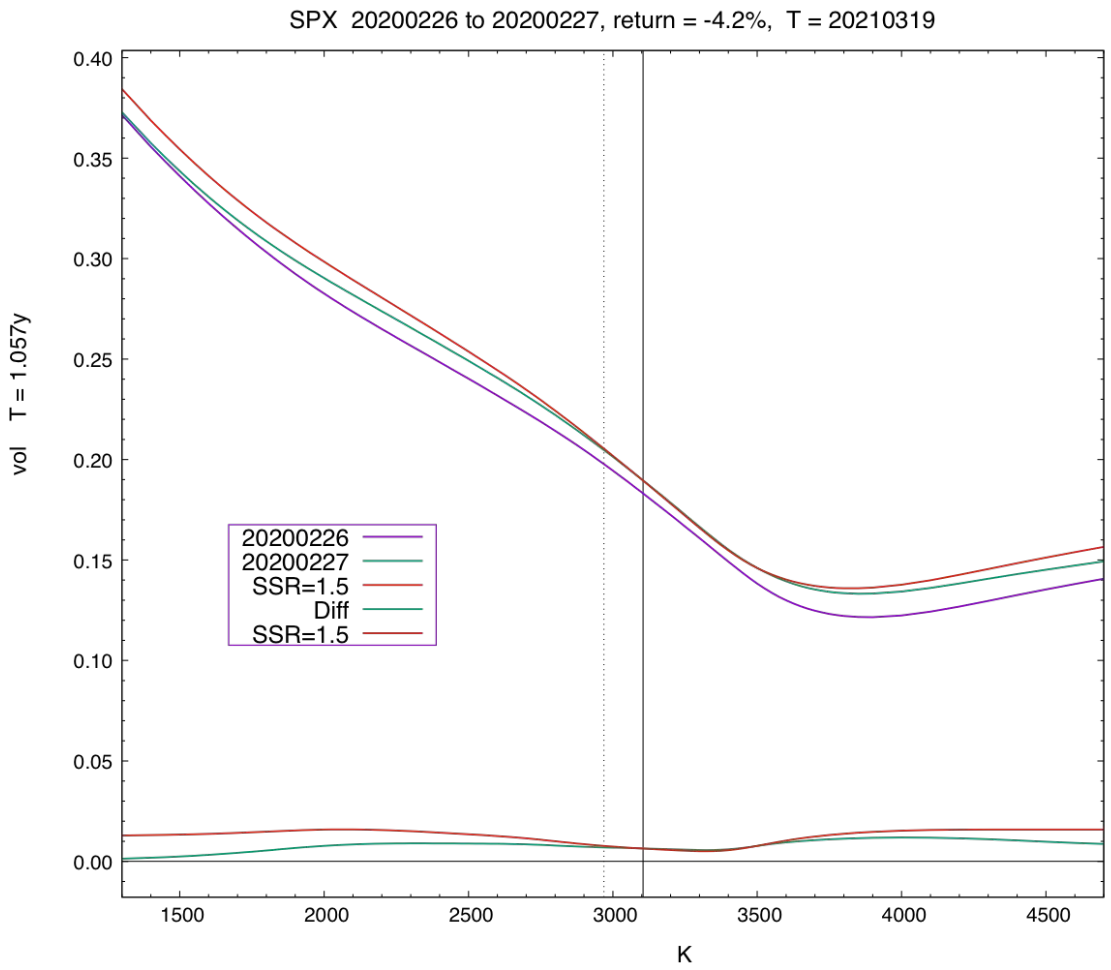
Close-to-close spot vol dynamics

SPX 2020-02-26 to 2020-02-27

Covid crash!

T = 2.5m, SSR = 2.0

Evidence for c2-spot-sensitivity > 0



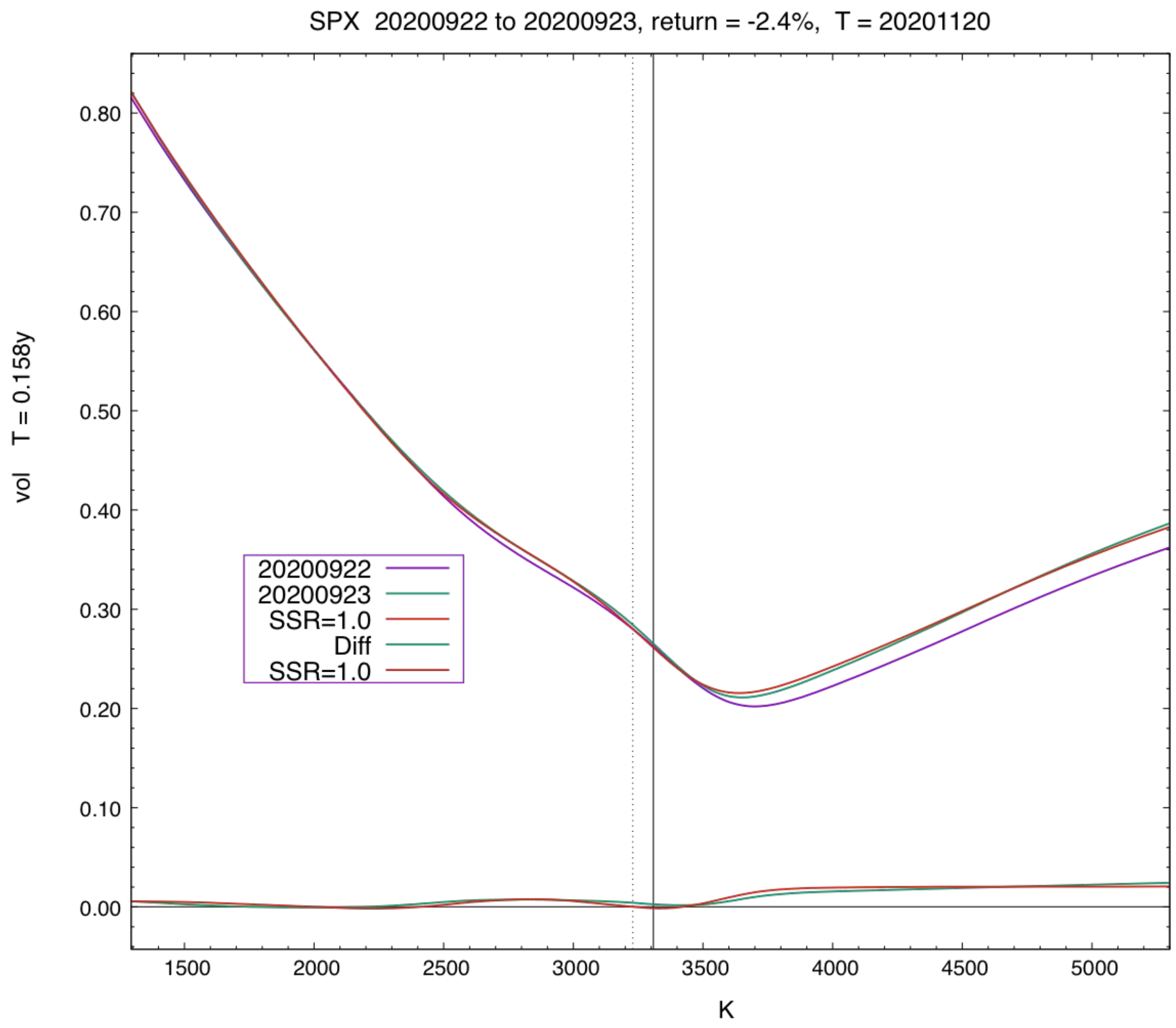
Close-to-close spot vol dynamics

SPX 2020-02-26 to 2020-02-27

Covid crash!

T = 1y, SSR = 1.5

Evidence for c2-spot-sensitivity > 0



Close-to-close spot vol dynamics

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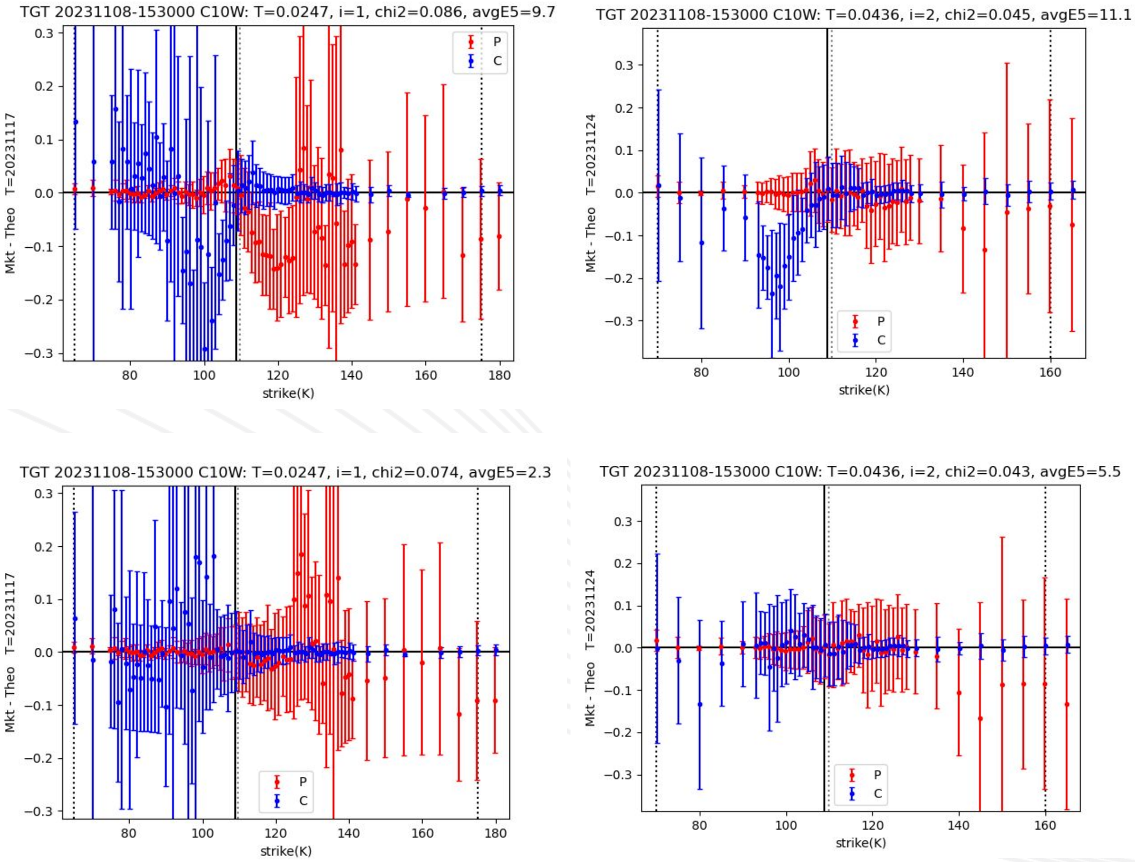
Subtleties of Pricing American “vanillas”

- In the olden days:
  - Could price every vanilla, European of American, with one flat r, q, and vol.
  - The same vol would work (well enough…) for call and put at same T,K.
- Already pretty hard, especially in real time. One needs:
  - A proper **cash dividend model** (no consensus even for vanilla…).
  - Handle **settlement** effects (incl. exchange and bank holidays).
  - A good choice of “**vol time**” (aka “business time”), including “**events**”.
    - NOTE: Pricing with vol time is equivalent to pricing with a (particular) vol term-structure.
  - Then: imply “SPIBOR” (~daily), borrows (real time), and vol surfaces (real time).
  - “American PCP” condition to imply borrow: Demand volP(K) = volC(K) around ATM

Subtleties of Pricing American "vanillas" 2

- Now: How fancy does the modeling have to be? ("De-Americanization")
  - BS: (1) Flat r,q, vol (2) r(t), q(t), vol (3) r(t), q(t), vol(t) for each K(?)
  - Beyond-BS: (4) r(t), q(t), LV, (5) r(t), q(t), SLV, (6) Other (approx/hacks...)
- Empirically in US: One definitely needs rate TS, vol-time including events, settlement, proper dividend modeling.
- In Europe: Evidence that local vols (or roughly equiv approx's) are being used.
- Let's look at some examples:
  - Rate TS and event effects: MSFT, TSLA, TGT
  - Settlement effects (+more): SPX

Event Time Effect on Pricing American Vanillas



TGT 2023-11-08

Target has a dividend and earnings call just before expiry T=2023-11-17 (i=1).

Top row: Without an "event time" an implied borrow allows (OTM and ITM) market prices to be matched at a few strikes, but not all.

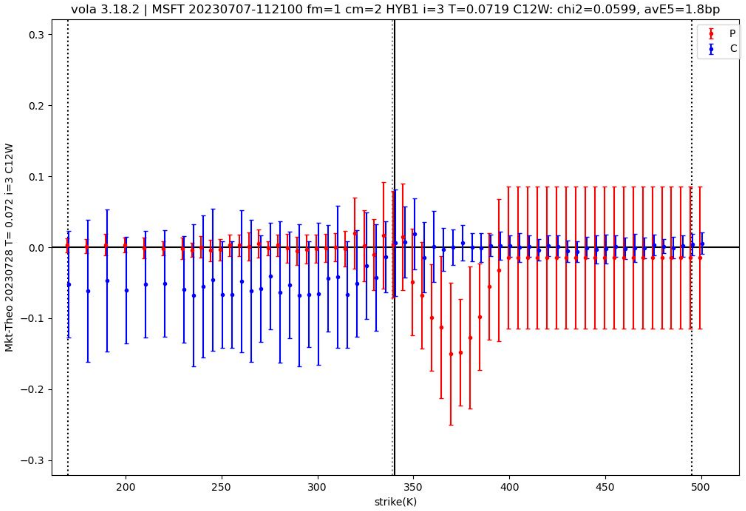
Bottom row: With an event time of 0.09y all prices can be matched, in all expiries!

Rate TS and Event Time for American Vanillas

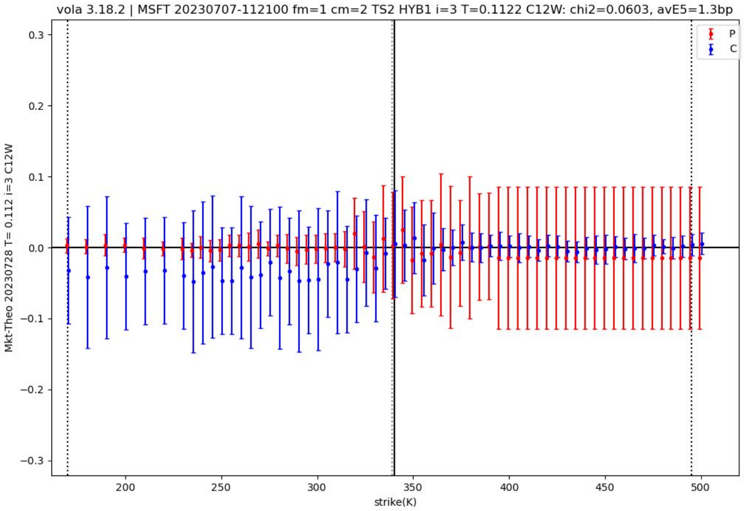
MSFT 2023-07-07

The ultimate test of a valuation approach is always the **price-difference plot**: Mkt - Theo

Flat term rates r(T), q(T)



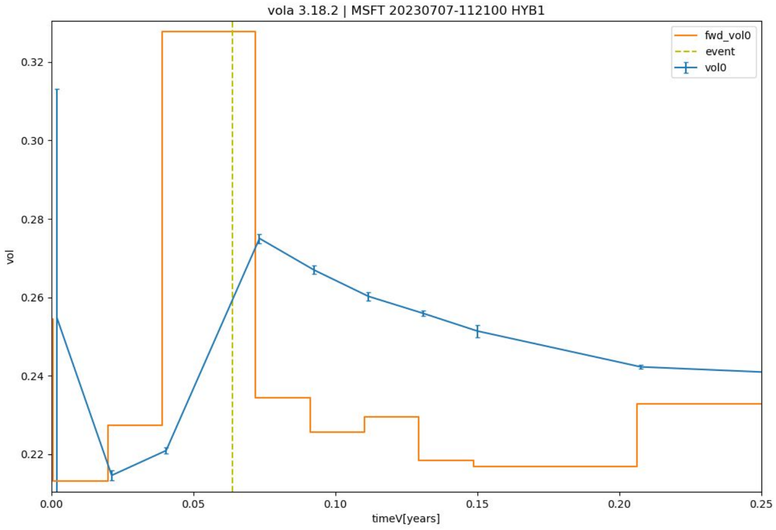
Local r(t),q(t) and $\Delta T_E = 0.04y$



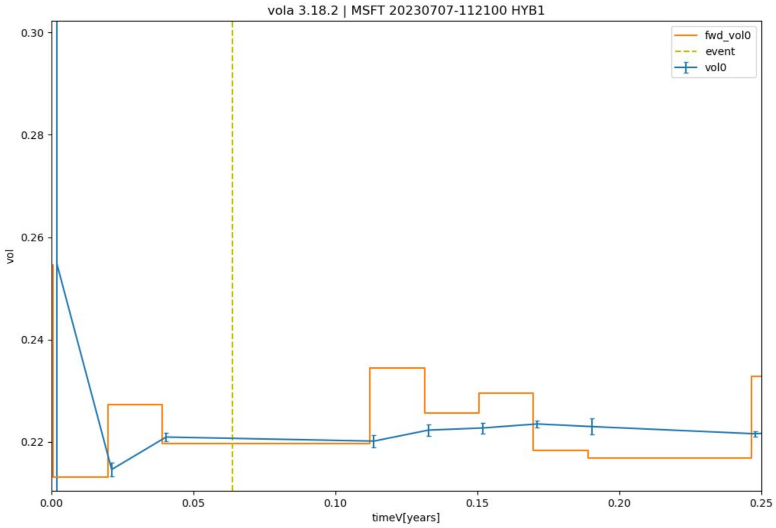
Rate TS and Event Time for American Vanillas

MSFT 2023-07-07

“Dirty” ATF vols

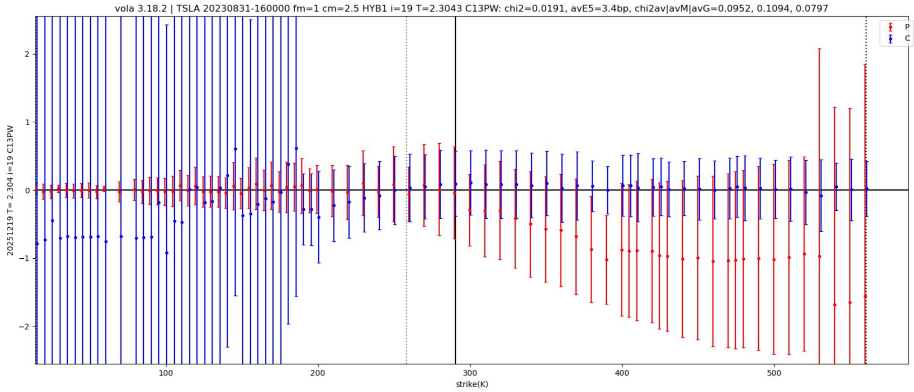


“Clean” ATF vols, $\Delta T_E = 0.04y$



Rate Term-Structure Effect on Pricing American Vanillas

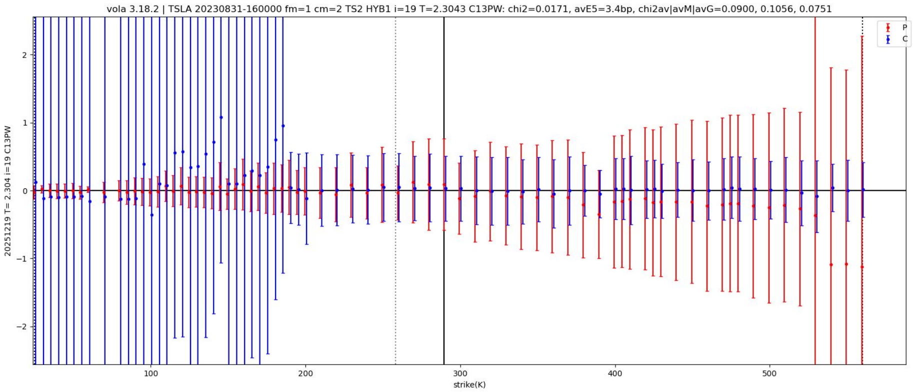
TSLA 2023-08-31



Price-Difference plot: Mkt - Theo

← Pricing with flat term r,q

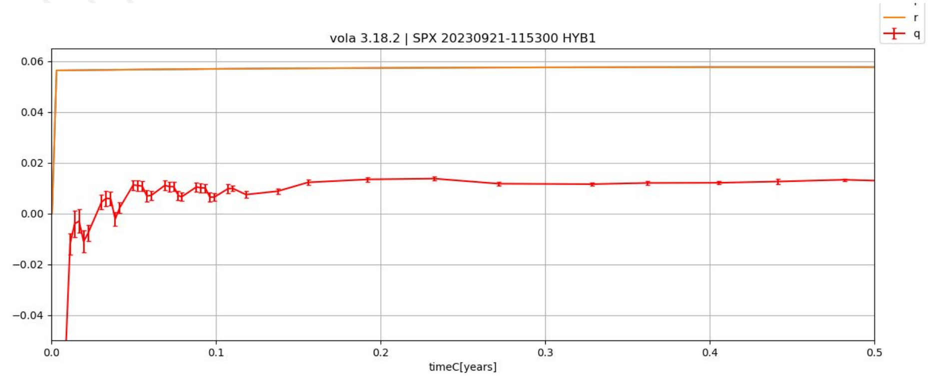
T = 2.4y



← Pricing with local r(t),q(t)

Settlement Effects for SPX options

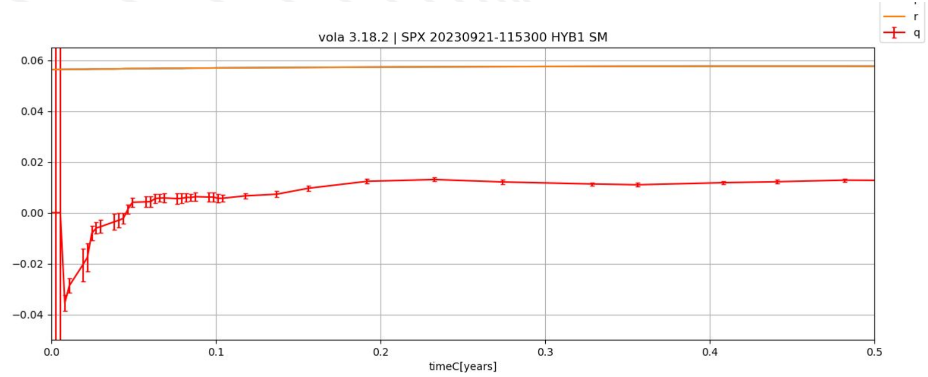
Let’s treat SPX like an equity with a “spot”, borrow cost, and (perhaps) cash dividends.



Implied borrow cost term structure

← Ignoring settlement, wrong spot

Wrong spot shows up as 1/T term in the borrow TS (made up wrong spot for illustration here…).



← With settlement, wrong spot

Now short-term borrow TS is smooth.

Settlement Effects for SPX options

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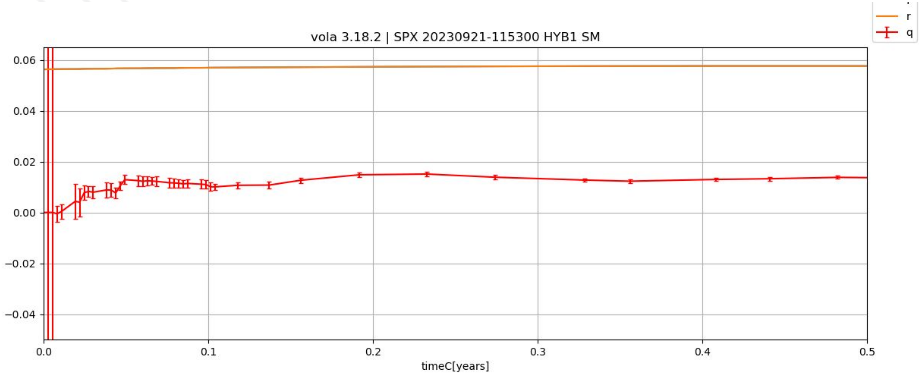
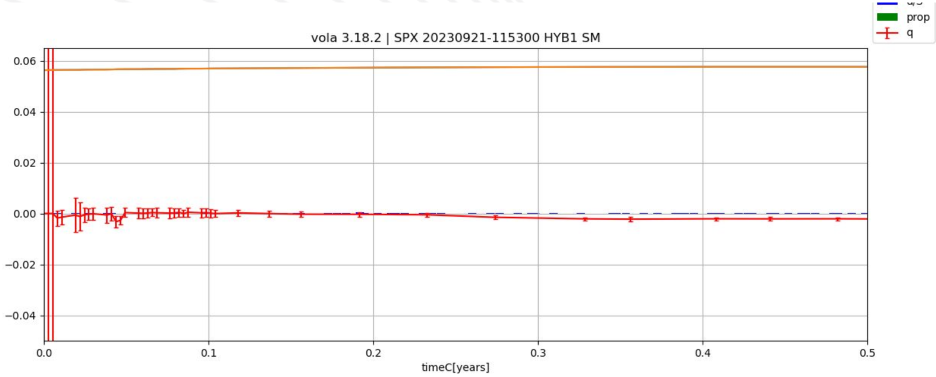


Implied borrow cost term structure

← With settlement, implied spot

No divs, so borrow includes div yield



← With settlement, implied spot

With divs, so borrow is “pure” and very flat close to 0

THE END